

9 Exponentiële en logaritmische functies

Voorkennis De logaritme

Bladzijde 9

- 1** a ${}^2\log(64) = {}^2\log(2^6) = 6$ d ${}^{\frac{1}{3}}\log(1) = {}^{\frac{1}{3}}\log((\frac{1}{3})^0) = 0$
 b ${}^3\log(\frac{1}{27}) = {}^3\log(3^{-3}) = -3$ e ${}^{\frac{1}{2}}\log(\frac{1}{8}) = {}^{\frac{1}{2}}\log((\frac{1}{2})^3) = 3$
 c ${}^6\log(6\sqrt{6}) = {}^6\log(6^{\frac{1}{2}}) = 1\frac{1}{2}$ f ${}^3\log(\frac{1}{9}\sqrt{3}) = {}^3\log(3^{-2} \cdot 3^{\frac{1}{2}}) = {}^3\log(3^{-\frac{3}{2}}) = -1\frac{1}{2}$
- 2** a ${}^3\log(x) = 5$ d ${}^{\frac{1}{2}}\log(3x + 4) = -2$
 $x = 3^5 = 243$ $3x + 4 = (\frac{1}{2})^{-2}$
 ${}^2\log(3x + 7) = 4$ $3x + 4 = 4$
 $3x + 7 = 2^4$ $3x = 0$
 $3x + 7 = 16$ $x = 0$
 $3x = 9$ e $3 + {}^2\log(5 - 1\frac{1}{2}x) = 0$
 $x = 3$ ${}^2\log(5 - 1\frac{1}{2}x) = -3$
 c $5 + {}^5\log(x + 3) = 7$ $5 - 1\frac{1}{2}x = 2^{-3}$
 ${}^5\log(x + 3) = 2$ $5 - 1\frac{1}{2}x = \frac{1}{8}$
 $x + 3 = 5^2$ $40 - 12x = 1$
 $x + 3 = 25$ $-12x = -39$
 $x = 22$ $x = 3\frac{1}{4}$
- f $4 - 2 \cdot {}^3\log(x) = 3$
 $-2 \cdot {}^3\log(x) = -1$
 ${}^3\log(x) = \frac{1}{2}$
 $x = 3^{\frac{1}{2}} = \sqrt{3}$

9.1 Rekenregels voor logaritmen

Bladzijde 10

- 1** a ${}^2\log(a)$ is de exponent van een macht met grondtal 2 waarmee de macht gelijk is aan a , dus $2^{{}^2\log(a)} = a$. Omdat een macht met grondtal 2 altijd groter is dan 0, geldt dit alleen voor $a > 0$.
 b ${}^g\log(x) = y$ oftewel $y = {}^g\log(x)$ substitueren in $x = g^y$ geeft $x = g^{{}^g\log(x)}$, dus $g^{{}^g\log(x)} = x$.

Bladzijde 11

- 2** $2 \cdot {}^2\log(800) = 2 \cdot {}^2\log(32 \cdot 25) = 2 \cdot ({}^2\log(32) + {}^2\log(25)) = 2 \cdot ({}^2\log(2^5) + {}^2\log(5^2)) = 2 \cdot (5 + 2 \cdot {}^2\log(5)) = 10 + 4 \cdot {}^2\log(5)$
- 3** a ${}^2\log(6) + {}^2\log(10) = {}^2\log(6 \cdot 10) = {}^2\log(60)$
 b ${}^3\log(30) - {}^3\log(6) = {}^3\log(\frac{30}{6}) = {}^3\log(5)$
 c $2 \cdot {}^5\log(3) + {}^5\log(\frac{1}{2}) = {}^5\log(3^2) + {}^5\log(\frac{1}{2}) = {}^5\log(9) + {}^5\log(\frac{1}{2}) = {}^5\log(9 \cdot \frac{1}{2}) = {}^5\log(4\frac{1}{2})$
 d ${}^{\frac{1}{2}}\log(15) - 4 \cdot {}^{\frac{1}{2}}\log(3) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(3^4) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(81) = {}^{\frac{1}{2}}\log(\frac{15}{81}) = {}^{\frac{1}{2}}\log(\frac{5}{27})$
 e $-2 \cdot {}^4\log(6) + {}^4\log(12) = {}^4\log(6^{-2}) + {}^4\log(12) = {}^4\log(\frac{1}{36}) + {}^4\log(12) = {}^4\log(\frac{1}{36} \cdot 12) = {}^4\log(\frac{1}{3})$
 f $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(50) - \log(25) = \log(\frac{50}{25}) = \log(2)$

- 4** a $4 + {}^2\log(3) = {}^2\log(2^4) + {}^2\log(3) = {}^2\log(16) + {}^2\log(3) = {}^2\log(16 \cdot 3) = {}^2\log(48)$
- b $3 - \frac{1}{2}\log(10) = \frac{1}{2}\log\left(\left(\frac{1}{2}\right)^3\right) - \frac{1}{2}\log(10) = \frac{1}{2}\log\left(\frac{1}{8}\right) - \frac{1}{2}\log(10) = \frac{1}{2}\log\left(\frac{1}{80}\right) = \frac{1}{2}\log\left(\frac{1}{80}\right)$
- c $2 - \log(5) = \log(10^2) - \log(5) = \log(100) - \log(5) = \log\left(\frac{100}{5}\right) = \log(20)$
- d ${}^2\log(12) - {}^3\log(9) = {}^2\log(12) - 2 = {}^2\log(12) - {}^2\log(2^2) = {}^2\log(12) - {}^2\log(4) = {}^2\log\left(\frac{12}{4}\right) = {}^2\log(3)$
- e $\frac{1}{2} \cdot {}^3\log(16) + \frac{1}{2}\log(8) = {}^3\log(16^{\frac{1}{2}}) - 3 = {}^3\log(4) - {}^3\log(3^3) = {}^3\log(4) - {}^3\log(27) = {}^3\log\left(\frac{4}{27}\right)$
- f $\log(500) - {}^5\log(125) = \log(500) - 3 = \log(500) - \log(10^3) = \log(500) - \log(1000) = \log\left(\frac{500}{1000}\right) = \log\left(\frac{1}{2}\right)$
- 5** a ${}^3\log(6) + {}^3\log\left(1\frac{1}{2}\right) = {}^3\log\left(6 \cdot 1\frac{1}{2}\right) = {}^3\log(9) = 2$
- b ${}^5\log(2) - {}^5\log(50) = {}^5\log\left(\frac{2}{50}\right) = {}^5\log\left(\frac{1}{25}\right) = {}^5\log(5^{-2}) = -2$
- c ${}^2\log(27) + 3 \cdot {}^2\log\left(\frac{1}{6}\right) = {}^2\log(27) + {}^2\log\left(\left(\frac{1}{6}\right)^3\right) = {}^2\log(27) + {}^2\log\left(\frac{1}{216}\right) = {}^2\log\left(\frac{27}{216}\right) = {}^2\log\left(\frac{1}{8}\right) = {}^2\log(2^{-3}) = -3$
- d $2 \cdot {}^4\log(6) - 2 \cdot {}^4\log(3) = {}^4\log(6^2) - {}^4\log(3^2) = {}^4\log(36) - {}^4\log(9) = {}^4\log\left(\frac{36}{9}\right) = {}^4\log(4) = 1$
- 6** a $\frac{g^{{}^g\log(a)} - g^{{}^g\log(b)}}{g^{{}^g\log(b)}} = \frac{a}{b} = g^{{}^g\log\left(\frac{a}{b}\right)}$, dus ${}^g\log(a) - {}^g\log(b) = {}^g\log\left(\frac{a}{b}\right)$.
- b $g^{p \cdot {}^g\log(a)} = (g^{{}^g\log(a)})^p = a^p = g^{{}^g\log(a^p)}$, dus $p \cdot {}^g\log(a) = {}^g\log(a^p)$.
- 7** a ${}^2\log(a) + 3 \cdot {}^2\log(b) = {}^2\log(a) + {}^2\log(b^3) = {}^2\log(ab^3)$
- b $5 \cdot {}^3\log(a) - 2 \cdot {}^3\log(b) = {}^3\log(a^5) - {}^3\log(b^2) = {}^3\log\left(\frac{a^5}{b^2}\right)$
- c $2 + {}^5\log(a) = {}^5\log(5^2) + {}^5\log(a) = {}^5\log(25) + {}^5\log(a) = {}^5\log(25a)$
- d $2 - {}^3\log(a) = {}^3\log(3^2) - {}^3\log(a) = {}^3\log(9) - {}^3\log(a) = {}^3\log\left(\frac{9}{a}\right)$
- e ${}^6\log(a) - 1 = {}^6\log(a) - {}^6\log(6) = {}^6\log\left(\frac{a}{6}\right) = {}^6\log\left(\frac{1}{6}a\right)$
- f $2 \cdot {}^5\log(b) + \frac{1}{2} \cdot {}^5\log(a) = {}^5\log(b^2) + {}^5\log(a^{\frac{1}{2}}) = {}^5\log(b^2) + {}^5\log(\sqrt{a}) = {}^5\log(b^2 \cdot \sqrt{a})$

Bladzijde 12

- 8** a $\log(600) = \log(100 \cdot 6) = \log(100) + \log(6) = 2 + \log(6)$
- b ${}^2\log(24) = {}^2\log(8 \cdot 3) = {}^2\log(8) + {}^2\log(3) = 3 + {}^2\log(3)$
- c $4 \cdot {}^3\log(54) = 4 \cdot {}^3\log(27 \cdot 2) = 4 \cdot ({}^3\log(27) + {}^3\log(2)) = 4 \cdot (3 + {}^3\log(2)) = 12 + 4 \cdot {}^3\log(2) = 12 + {}^3\log(2^4) = 12 + {}^3\log(16)$
- d $2 \cdot {}^5\log(1250) = 2 \cdot {}^5\log(625 \cdot 2) = 2 \cdot ({}^5\log(625) + {}^5\log(2)) = 2 \cdot (4 + {}^5\log(2)) = 8 + 2 \cdot {}^5\log(2) = 8 + {}^5\log(2^2) = 8 + {}^5\log(4)$
- e $\log(1600) - 1 = \log(100 \cdot 16) - 1 = \log(100) + \log(16) - 1 = 2 + \log(16) - 1 = 1 + \log(16)$
- f $7 + {}^2\log(1600) = 7 + {}^2\log(64 \cdot 25) = 7 + {}^2\log(64) + {}^2\log(25) = 7 + 6 + {}^2\log(25) = 13 + {}^2\log(25)$
- 9** a $y = {}^3\log(x) \xrightarrow{\text{verm. } y\text{-as, } 81} y = {}^3\log\left(\frac{1}{81}x\right)$
 Er geldt ${}^3\log\left(\frac{1}{81}x\right) = {}^3\log\left(\frac{1}{81}\right) + {}^3\log(x) = -4 + {}^3\log(x)$.
 Dus $a = -4$.
- b Je had de grafiek van g ook kunnen krijgen met de translatie $(0, -4)$.
- 10** a $y = {}^2\log(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{32}} y = {}^2\log(32x)$
 Er geldt ${}^2\log(32x) = {}^2\log(32) + {}^2\log(x) = 5 + {}^2\log(x)$.
 Dus de translatie $(0, 5)$ levert dezelfde beeldgrafiek op.
- b $y = {}^2\log(x) \xrightarrow{\text{translatie } (0, 3)} y = {}^2\log(x) + 3$
 Er geldt ${}^2\log(x) + 3 = {}^2\log(x) + {}^2\log(2^3) = {}^2\log(x) + {}^2\log(8) = {}^2\log(8x)$
 Dus de vermenigvuldiging ten opzichte van de y -as met $\frac{1}{8}$ levert dezelfde beeldgrafiek op.

$$11 \quad y = {}^2\log(x) \xrightarrow{\text{translatie } (3, 0)} y = {}^2\log(x-3) \xrightarrow{\text{verm. } y \rightarrow ay, \frac{1}{4}} y = {}^2\log(4x-3)$$

$$\text{Er geldt } {}^2\log(4x-3) = {}^2\log(4(x-\frac{3}{4})) = {}^2\log(4) + {}^2\log(x-\frac{3}{4}) = 2 + {}^2\log(x-\frac{3}{4}).$$

$$\text{Dus } p = 2 \text{ en } q = -\frac{3}{4}.$$

$$12 \quad y = 3^x \xrightarrow{\text{translatie } (a, 0)} y = 3^{x-a} \xrightarrow{\text{verm. } y \rightarrow ay, b} y = 3^{\frac{1}{b}x-a}$$

$$g(x) = 81 \cdot 27^x = 3^4 \cdot (3^3)^x = 3^4 \cdot 3^{3x} = 3^{3x+4}$$

$$\text{Dus } b = \frac{1}{3} \text{ en } a = -4.$$

$$y = {}^3\log(x) \xrightarrow{\text{translatie } (-4, 0)} y = {}^3\log(x+4) \xrightarrow{\text{verm. } y \rightarrow ay, \frac{1}{3}} y = {}^3\log(3x+4)$$

$$\text{Er geldt } {}^3\log(3x+4) = {}^3\log(3(x+\frac{4}{3})) = {}^3\log(3) + {}^3\log(x+\frac{4}{3}) = 1 + {}^3\log(x+\frac{4}{3}).$$

$$\text{Dus } p = 1 \text{ en } q = \frac{4}{3}.$$

$$13 \quad \text{a} \quad 3 + {}^2\log(3) = {}^2\log(2^3) + {}^2\log(3) = {}^2\log(8) + {}^2\log(3) = {}^2\log(8 \cdot 3) = {}^2\log(24)$$

$$\text{b} \quad {}^2\log(x+1) = 3 + {}^2\log(3)$$

$${}^2\log(x+1) = {}^2\log(24)$$

$$x+1 = 24$$

$$x = 23$$

Bladzijde 14

$$14 \quad \text{a} \quad {}^5\log(x) = 3 \cdot {}^5\log(2) - 2 \cdot {}^5\log(3)$$

$${}^5\log(x) = {}^5\log(2^3) - {}^5\log(3^2)$$

$${}^5\log(x) = {}^5\log(8) - {}^5\log(9)$$

$${}^5\log(x) = {}^5\log(\frac{8}{9})$$

$$x = \frac{8}{9}$$

$$\text{b} \quad {}^2\log(x) = 4 - {}^2\log(3)$$

$${}^2\log(x) = {}^2\log(2^4) - {}^2\log(3)$$

$${}^2\log(x) = {}^2\log(16) - {}^2\log(3)$$

$${}^2\log(x) = {}^2\log(\frac{16}{3})$$

$$x = \frac{16}{3}$$

$$\text{c} \quad {}^2\log(x+3) = 3 + {}^2\log(x)$$

$${}^2\log(x+3) = {}^2\log(2^3) + {}^2\log(x)$$

$${}^2\log(x+3) = {}^2\log(8) + {}^2\log(x)$$

$${}^2\log(x+3) = {}^2\log(8x)$$

$$x+3 = 8x$$

$$-7x = -3$$

$$x = \frac{3}{7}$$

$$\text{d} \quad {}^3\log(2x) = 1 + {}^3\log(x+1)$$

$${}^3\log(2x) = {}^3\log(3) + {}^3\log(x+1)$$

$${}^3\log(2x) = {}^3\log(3(x+1))$$

$${}^3\log(2x) = {}^3\log(3x+3)$$

$$2x = 3x+3$$

$$-x = 3$$

$$x = -3$$

$$\text{vold. niet}$$

$$15 \quad \text{a} \quad 5 \cdot \log(x) = 5 - \log(3125)$$

$$5 \cdot \log(x) = 5 - \log(5^5)$$

$$5 \cdot \log(x) = 5 - 5 \cdot \log(5)$$

$$\log(x) = 1 - \log(5)$$

$$\log(x) = \log(10) - \log(5)$$

$$\log(x) = \log(\frac{10}{5})$$

$$x = 2$$

$$\text{b} \quad {}^{\frac{1}{2}}\log(2x-1) = 2 + {}^{\frac{1}{2}}\log(x+2)$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(x+2)$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}) + {}^{\frac{1}{2}}\log(x+2)$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}(x+2))$$

$${}^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4}x + \frac{1}{2})$$

$$2x-1 = \frac{1}{4}x + \frac{1}{2}$$

$$1\frac{3}{4}x = 1\frac{1}{2}$$

$$x = \frac{6}{7}$$

$$\text{c} \quad {}^3\log(x+2) = 1 - {}^3\log(x)$$

$${}^3\log(x+2) = {}^3\log(3) - {}^3\log(x)$$

$${}^3\log(x+2) = {}^3\log(\frac{3}{x})$$

$$x+2 = \frac{3}{x}$$

$$x^2 + 2x = 3$$

$$x^2 + 2x - 3 = 0$$

$$(x-1)(x+3) = 0$$

$$x = 1 \vee x = -3$$

$$\text{vold. niet}$$

$$\text{d} \quad 2 \cdot {}^3\log(x) + 1 = {}^3\log(5x-2)$$

$${}^3\log(x^2) + {}^3\log(3) = {}^3\log(5x-2)$$

$${}^3\log(3x^2) = {}^3\log(5x-2)$$

$$3x^2 = 5x-2$$

$$3x^2 - 5x + 2 = 0$$

$$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 1$$

$$x = \frac{5+1}{6} = 1 \vee x = \frac{5-1}{6} = \frac{2}{3}$$

16 a ${}^5\log(x) = 2 + \frac{1}{2} \cdot {}^5\log(3)$
 ${}^5\log(x) = {}^5\log(5^2) + {}^5\log(3^{\frac{1}{2}})$
 ${}^5\log(x) = {}^5\log(25) + {}^5\log(\sqrt{3})$
 ${}^5\log(x) = {}^5\log(25\sqrt{3})$
 $x = 25\sqrt{3}$

b ${}^3\log(x+4) + 1 = 2 \cdot {}^3\log(x-2)$
 ${}^3\log(x+4) + {}^3\log(3) = {}^3\log((x-2)^2)$
 ${}^3\log(3(x+4)) = {}^3\log(x^2 - 4x + 4)$
 ${}^3\log(3x+12) = {}^3\log(x^2 - 4x + 4)$
 $3x+12 = x^2 - 4x + 4$
 $x^2 - 7x - 8 = 0$
 $(x+1)(x-8) = 0$
 $x = -1 \vee x = 8$
 vold. niet

c ${}^2\log(2x) - {}^2\log(x+3) = {}^2\log(x) - 2$
 ${}^2\log\left(\frac{2x}{x+3}\right) = {}^2\log(x) - {}^2\log(2^2)$
 ${}^2\log\left(\frac{2x}{x+3}\right) = {}^2\log\left(\frac{x}{4}\right)$

$$\frac{2x}{x+3} = \frac{x}{4}$$

$$x^2 + 3x = 8x$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x = 0 \vee x = 5$$

vold. niet

d ${}^3\log(x) = 2 - {}^3\log(x-1)$
 ${}^3\log(x) = {}^3\log(3^2) - {}^3\log(x-1)$
 ${}^3\log(x) = {}^3\log\left(\frac{9}{x-1}\right)$

$$x = \frac{9}{x-1}$$

$$x^2 - x = 9$$

$$x^2 - x - 9 = 0$$

$$D = (-1)^2 - 4 \cdot 1 \cdot -9 = 37$$

$$x = \frac{1 + \sqrt{37}}{2} \vee x = \frac{1 - \sqrt{37}}{2}$$

$$x = \frac{1}{2} + \frac{1}{2}\sqrt{37} \vee x = \frac{1}{2} - \frac{1}{2}\sqrt{37}$$

vold. niet

Bladzijde 15

17 $y = {}^2\log(x) \xrightarrow{\text{verm. } x\text{-as, } a} y = a \cdot {}^2\log(x) \xrightarrow{\text{verm. } y\text{-as, } b} y = a \cdot {}^2\log\left(\frac{1}{b}x\right)$

Er geldt $a \cdot {}^2\log\left(\frac{1}{b}x\right) = a \cdot ({}^2\log(x) - {}^2\log(b)) = -a \cdot {}^2\log(b) + a \cdot {}^2\log(x)$.

Dus $a = 10$ en $-10 \cdot {}^2\log(b) = -5$

$${}^2\log(b) = \frac{1}{2}$$

$$b = 2^{\frac{1}{2}} = \sqrt{2}$$

18 a $p^{\log(g)} = g$
 $(p^{\log(g)})^{\log(a)} = g^{\log(a)}$
 $p^{\log(g) \cdot \log(a)} = g^{\log(a)}$
 $p^{\log(g) \cdot \log(a)} = a$

b $p^{\log(g) \cdot \log(a)} = p^{\log(a)}$
 $\log(g) \cdot \log(a) = \log(a)$
 $\log(a) = \frac{\log(a)}{\log(g)}$

Bladzijde 16

19 a Omdat is gegeven dat ${}^4\log(x) = {}^2\log(x-2)$ zal gelden dat $a = b$ als ${}^4\log(x) = a$ en

$${}^2\log(x-2) = b.$$

Substitutie van ${}^4\log(x) = a$ en ${}^2\log(x-2) = b$ in ${}^4\log(x) = {}^2\log(x-2)$ geeft $a = b$.

Uit ${}^4\log(x) = y$ geeft $x = g^y$ volgt ${}^4\log(x) = a$ geeft $x = 4^a$ en ${}^2\log(x-2) = b$ geeft $x-2 = 2^b$ oftewel $x = 2^b + 2$.

b Uit $x = 4^a$, $x = 2^b + 2$ en $a = b$ volgt $4^a = 2^a + 2$

$$(2^2)^a = 2^a + 2$$

$$(2^a)^2 - 2^a - 2 = 0$$

$$\text{Stel } 2^a = u.$$

$$u^2 - u - 2 = 0$$

$$(u + 1)(u - 2) = 0$$

$$u = -1 \vee u = 2$$

$$2^a = -1 \vee 2^a = 2$$

$$a = 1$$

$a = 1$ geeft $x = 4$

Bladzijde 17

20 a ${}^3\log(3x - 5) + {}^{\frac{1}{3}}\log(x - 1) = 0$
 ${}^3\log(3x - 5) - {}^3\log(x - 1) = 0$
 ${}^3\log(3x - 5) = {}^3\log(x - 1)$

$$3x - 5 = x - 1$$

$$2x = 4$$

$$x = 2$$

b ${}^5\log(3x) + 2 \cdot {}^{\frac{1}{5}}\log(x) = 0$

$${}^5\log(3x) + {}^{\frac{1}{5}}\log(x^2) = 0$$

$${}^5\log(3x) - {}^5\log(x^2) = 0$$

$${}^5\log(3x) = {}^5\log(x^2)$$

$$3x = x^2$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0 \vee x = 3$$

vold. niet

c $2x \cdot {}^{\frac{1}{3}}\log(3x + 5) = {}^{\frac{1}{3}}\log(3x + 5)$

$${}^{\frac{1}{3}}\log(3x + 5) = 0 \vee 2x = 1$$

$$3x + 5 = 1 \vee x = \frac{1}{2}$$

$$3x = -4 \vee x = \frac{1}{2}$$

$$x = -1\frac{1}{3} \vee x = \frac{1}{2}$$

d ${}^2\log(x) = {}^4\log(x + 20)$

$${}^2\log(x) = \frac{{}^2\log(x + 20)}{{}^2\log(4)}$$

$${}^2\log(x) = \frac{{}^2\log(x + 20)}{2}$$

$$2 \cdot {}^2\log(x) = {}^2\log(x + 20)$$

$${}^2\log(x^2) = {}^2\log(x + 20)$$

$$x^2 = x + 20$$

$$x^2 - x - 20 = 0$$

$$(x + 4)(x - 5) = 0$$

$$x = -4 \vee x = 5$$

vold. niet

21 a $-2 \cdot {}^{\frac{1}{2}}\log(x) = 2 + {}^2\log(3 - x)$
 $2 \cdot {}^2\log(x) = {}^2\log(2^2) + {}^2\log(3 - x)$
 ${}^2\log(x^2) = {}^2\log(4(3 - x))$

$$x^2 = 12 - 4x$$

$$x^2 + 4x - 12 = 0$$

$$(x - 2)(x + 6) = 0$$

$$x = 2 \vee x = -6$$

vold. niet

b ${}^9\log(2x) = {}^3\log(x - 4)$

$$\frac{{}^3\log(2x)}{{}^3\log(9)} = {}^3\log(x - 4)$$

$$\frac{{}^3\log(2x)}{2} = {}^3\log(x - 4)$$

$${}^3\log(2x) = 2 \cdot {}^3\log(x - 4)$$

$${}^3\log(2x) = {}^3\log((x - 4)^2)$$

$$2x = (x - 4)^2$$

$$2x = x^2 - 8x + 16$$

$$x^2 - 10x + 16 = 0$$

$$(x - 2)(x - 8) = 0$$

$$x = 2 \vee x = 8$$

vold. niet

c $4x \cdot {}^4\log(2x - 1) + 3 \cdot {}^4\log(2x - 1) = 0$
 $(4x + 3) \cdot {}^4\log(2x - 1) = 0$

$$4x + 3 = 0 \vee {}^4\log(2x - 1) = 0$$

$$4x = -3 \vee 2x - 1 = 1$$

$$x = -\frac{3}{4} \vee 2x = 2$$

$$x = -\frac{3}{4} \vee x = 1$$

vold. niet

d $x^2 \cdot {}^5\log(2x + 1) + 9 \cdot {}^{\frac{1}{5}}\log(2x + 1) = 0$

$$x^2 \cdot {}^5\log(2x + 1) - 9 \cdot {}^5\log(2x + 1) = 0$$

$$(x^2 - 9) \cdot {}^5\log(2x + 1) = 0$$

$$x^2 - 9 = 0 \vee {}^5\log(2x + 1) = 0$$

$$x^2 = 9 \vee 2x + 1 = 1$$

$$x = 3 \vee x = -3 \vee 2x = 0$$

$$x = 3 \vee x = -3 \vee x = 0$$

vold. niet

22 ${}^3\log^2(x) = 2 \cdot {}^3\log(x) + 15$

Stel ${}^3\log(x) = u$.

$u^2 = 2u + 15$

$u^2 - 2u - 15 = 0$

$(u + 3)(u - 5) = 0$

$u = -3 \vee u = 5$

${}^3\log(x) = -3 \vee {}^3\log(x) = 5$

$x = 3^{-3} = \frac{1}{27} \vee x = 3^5 = 243$

23 a ${}^2\log^2(x) = 2 \cdot {}^2\log(x) + 3$

Stel ${}^2\log(x) = u$.

$u^2 = 2u + 3$

$u^2 - 2u - 3 = 0$

$(u + 1)(u - 3) = 0$

$u = -1 \vee u = 3$

${}^2\log(x) = -1 \vee {}^2\log(x) = 3$

$x = \frac{1}{2} \vee x = 8$

b $\frac{1}{2}\log^2(x+2) + 3 \cdot \frac{1}{2}\log(x+2) = 0$

Stel $\frac{1}{2}\log(x+2) = u$.

$u^2 + 3u = 0$

$u(u + 3) = 0$

$u = 0 \vee u = -3$

$\frac{1}{2}\log(x+2) = 0 \vee \frac{1}{2}\log(x+2) = -3$

$x + 2 = 1 \vee x + 2 = 8$

$x = -1 \vee x = 6$

c $2 \cdot {}^3\log^2(x) + 2 = 5 \cdot {}^3\log(x)$

Stel ${}^3\log(x) = u$.

$2u^2 + 2 = 5u$

$2u^2 - 5u + 2 = 0$

$D = (-5)^2 - 4 \cdot 2 \cdot 2 = 9$

$u = \frac{5+3}{4} = 2 \vee u = \frac{5-3}{4} = \frac{1}{2}$

${}^3\log(x) = 2 \vee {}^3\log(x) = \frac{1}{2}$

$x = 9 \vee x = \sqrt[3]{3}$

d ${}^5\log^2(x) + 3 \cdot \frac{1}{5}\log(x) + 2 = 0$

${}^5\log^2(x) - 3 \cdot \frac{1}{5}\log(x) + 2 = 0$

Stel ${}^5\log(x) = u$.

$u^2 - 3u + 2 = 0$

$(u - 1)(u - 2) = 0$

$u = 1 \vee u = 2$

${}^5\log(x) = 1 \vee {}^5\log(x) = 2$

$x = 5 \vee x = 25$

9.2 Exponentiële en logaritmische formules

Bladzijde 19

- 24 a** Jaarlijkse toename van 5%, dus de vermenigvuldigingsfactor is 1,05.
Het bedrag op 1 januari 2021 is $1000 \cdot 1,05 = 1050$ euro.
Het bedrag op 1 januari 2025 is $1000 \cdot 1,05^5 \approx 1276,28$ euro.
- b** $g = 1,05$
- c** procentuele toename $\approx \frac{1276,28 - 1000}{1000} \cdot 100\% \approx 27,6\%$

Bladzijde 21

- 25 a** $g_{\text{jaar}} = 1,127$
- b** $g_{\text{maand}} = 0,932$
- c** Het groeipercentage per maand is 73,5%.
- d** De afname per dag is 15,5%.
- e** Het groeipercentage per jaar is 142%.
- f** $g_{\text{dag}} = 0,993$

- 26 a** $g_{\text{kwartier}} = 1,12$
 $g_{\text{uur}} = 1,12^4 \approx 1,574$
De toename is 57,4% per uur.
- b** $g_{\text{kwartier}} = 1,12$
 $g_{5 \text{ minuten}} = 1,12^{\frac{1}{3}} \approx 1,038$
De procentuele toename per vijf minuten is 3,8%.
- c** $g_{\text{kwartier}} = 1,12$
 $g_{5 \text{ uur}} = 1,12^{20} \approx 9,65$
De toename is 865% per vijf uur.

- 27 a** $g_{\text{dag}} = 1,3$
 $g_{\text{week}} = 1,3^7 \approx 6,27$
 Het groeipcentage per week is 527%.
- b** $g_{\text{dag}} = 1,3$
 $g_{4 \text{ uur}} = 1,3^{\frac{1}{4}} \approx 1,045$
 Het groeipcentage per vier uur is 4,5%.

- 28 a** $g_{\text{dag}} = 0,84$
 $g_{\text{week}} = 0,84^7 \approx 0,295$
- b** $g_{\text{dag}} = 0,84$
 $g_{\text{uur}} = 0,84^{\frac{1}{24}} \approx 0,993$
 De hoeveelheid neemt met 0,7% per uur af.
- c** $g_{\text{dag}} = 0,84$
 $g_{\text{kwartier}} = 0,84^{\frac{1}{24 \cdot 4}} \approx 0,9982$
 Het groeipcentage per kwartier is -0,18%.

Bladzijde 22

- 29 a** $g_{\text{uur}} = 0,805$
 $g_{\text{kwartier}} = 0,805^{\frac{1}{4}} \approx 0,947$
 De afname is 5,3% per kwartier.
- b** $g_{\text{jaar}} = 1,086$
 $g_{25 \text{ jaar}} = 1,086^{25} \approx 7,87$
 De toename is 687% per 25 jaar.
- c** $g_{\text{week}} = 2,80$
 $g_{\text{dag}} = 2,80^{\frac{1}{7}} \approx 1,158$
- 30 a** De hoeveelheid N neemt in $10 - 3 = 7$ uur toe van 1600 tot 4100, dus $g_{7 \text{ uur}} = \frac{4100}{1600}$.
- b** $g_{\text{uur}} = \left(\frac{4100}{1600}\right)^{\frac{1}{7}} = 1,1438\dots$
- c**
$$\left. \begin{aligned} N &= b \cdot 1,1438\dots^t \\ t &= 3 \text{ en } N = 1600 \end{aligned} \right\} \begin{aligned} b \cdot 1,1438\dots^3 &= 1600 \\ b &= \frac{1600}{1,1438\dots^3} \approx 1070 \end{aligned}$$

 Dus op $t = 0$ is $N \approx 1070$.
 De formule van N is $N = 1070 \cdot 1,144^t$.
- 31 a** Stel $N = b \cdot g^t$.
 $g_{6 \text{ uur}} = \frac{1250}{150} = 8,333\dots$
 $g_{\text{uur}} = 8,333\dots^{\frac{1}{6}} = 1,4238\dots$

$$\left. \begin{aligned} N &= b \cdot 1,4238\dots^t \\ t &= 2 \text{ en } N = 150 \end{aligned} \right\} \begin{aligned} b \cdot 1,4238\dots^2 &= 150 \\ b &= \frac{150}{1,4238\dots^2} \approx 74 \end{aligned}$$

 Dus $N = 74 \cdot 1,424^t$.
- b** Stel $H = b \cdot g^t$.
 $g_{3 \text{ dagen}} = \frac{0,47}{0,60} = 0,7833\dots$
 $g_{\text{dag}} = 0,7833\dots^{\frac{1}{3}} = 0,9218\dots$

$$\left. \begin{aligned} H &= b \cdot 0,9218\dots^t \\ t &= 5 \text{ en } H = 0,60 \end{aligned} \right\} \begin{aligned} b \cdot 0,9218\dots^5 &= 0,60 \\ b &= \frac{0,60}{0,9218\dots^5} = 0,901\dots \end{aligned}$$

 Dus $H = 0,90 \cdot 0,922^t$.

32 Stel $A = b \cdot g^t$.

$$g_{4 \text{ dagen}} = \frac{11}{31} = 0,3548\dots$$

$$g_{\text{dag}} = 0,3548\dots^{\frac{1}{4}} = 0,7718\dots$$

$$\left. \begin{array}{l} A = b \cdot 0,7718\dots^t \\ t = 3 \text{ en } A = 31 \end{array} \right\} b \cdot 0,7718\dots^3 = 31$$

$$b = \frac{31}{0,7718\dots^3} = 67,4\dots$$

$$60 \text{ uur} = \frac{60}{24} \text{ dag} = 2,5 \text{ dag}$$

$$t = 2,5 \text{ geeft } A = 67,4\dots \cdot 0,7718\dots^{2,5} \approx 35$$

Na 60 uur was de oppervlakte van de wond ongeveer 35 mm^2 .

33 a Voer in $y_1 = 17,0 \cdot 1,031^x$ en $y_2 = 34,0$.

De optie snijpunt geeft $x = 22,70\dots$

Na 22,7 jaar is het aantal verdubbeld.

b Voer in $y_1 = 12,9 \cdot 1,031^x$ en $y_2 = 25,8$.

De optie snijpunt geeft $x = 22,70\dots$

Na 22,7 jaar is het aantal verdubbeld.

Bladzijde 23

34 a $g_{\text{jaar}} = 1,131$

$$1,131^T = 2$$

$$T = {}^{1,131}\log(2) = 5,630\dots$$

De verdubbelingstijd is 5 jaar en $0,630\dots \cdot 12 \approx 8$ maanden.

b $g_{\text{week}} = 0,915$

$$0,915^T = \frac{1}{2}$$

$$T = {}^{0,915}\log\left(\frac{1}{2}\right) = 7,802\dots$$

De halveringstijd is 7 weken en $0,802\dots \cdot 7 \approx 6$ dagen.

Bladzijde 24

35 a $g_{\text{jaar}} = 1,0325$

$$1,0325^T = 2$$

$$T = {}^{1,0325}\log(2) = 21,6\dots$$

De verdubbelingstijd is 22 jaar.

b $g_{10 \text{ jaar}} = 1,081$

$$1,081^T = 2$$

$$T = {}^{1,081}\log(2) = 8,89\dots$$

De verdubbelingstijd is $8,89\dots \cdot 10 \text{ jaar} \approx 89 \text{ jaar}$.

36 a $g_{25 \text{ jaar}} = 2$

$$g_{\text{jaar}} = 2^{\frac{1}{25}} \approx 1,028$$

Het groeipercentage per jaar is 2,8%.

b $g_{28 \text{ jaar}} = \frac{1}{2}$

$$g_{\text{jaar}} = \left(\frac{1}{2}\right)^{\frac{1}{28}} \approx 0,976$$

De hoeveelheid radioactieve stof neemt per jaar met 2,4% af.

37 a $g_{\text{dag}} = 0,917$

$$0,917^T = \frac{1}{2}$$

$$T = {}^{0,917}\log\left(\frac{1}{2}\right) = 7,9\dots$$

De halveringstijd is 8 dagen.

b $g_{\text{dag}} = 0,917$

$$0,917^T = 0,1$$

$$T = {}^{0,917}\log(0,1) = 26,5\dots$$

Na 27 dagen is nog 10% van de beginhoeveelheid over.

- 38 a** $g_{\text{dag}} = 0,81$
 $g_{\text{week}} = 0,81^7 = 0,2287...$
 De afname per week is 77,1%.
- b** $g_{\text{week}} = 0,38$
 $g_{\text{dag}} = 0,38^{\frac{1}{7}} = 0,8709...$
 De afname per dag is 12,9%.
- c** De groeifactor per dag is 0,845.
 Dus $BZV = 300 \cdot 0,845^t$.
- d** $0,845^T = \frac{1}{2}$
 $T = {}^{0,845}\log(\frac{1}{2}) = 4,115...$
 De halveringstijd is 4 dagen en $0,115... \cdot 24 \approx 3$ uur.
- e** $300 \cdot 0,845^t = 10$
 $0,845^t = \frac{10}{300} = \frac{1}{30}$
 $t = {}^{0,845}\log(\frac{1}{30}) = 20,1...$
 Dus na 20 dagen zuiveren is het BZV afgenomen tot 10 mg/liter.
- f** $T = 10$ geeft $h = 7,6 \cdot 0,96^{10} = 5,0527...$
 Bij 10 °C is de groeifactor per dag 0,8709... (zie vraag b).
 $0,8709...^{5,0527...} \approx 0,5$, dus de formule klopt voor $T = 10$.
 $T = 20$ geeft $h = 7,6 \cdot 0,96^{20} = 3,3592...$
 Bij 20 °C is de groeifactor per dag 0,81 (zie vraag a).
 $0,81^{3,3592...} \approx 0,5$, dus de formule klopt voor $T = 20$.

Bladzijde 25

- 39** $y = 2^{x-1}$
 $2^{x-1} = y$
 $x - 1 = {}^2\log(y)$
 $x = 1 + {}^2\log(y)$

Bladzijde 26

- 40 a** $t = 1 + {}^2\log(0,2N)$
 $t = {}^2\log(2) + {}^2\log(0,2N)$
 $t = {}^2\log(0,4N)$
 $t = \frac{\log(0,4N)}{\log(2)}$
 $t \approx 3,32 \cdot \log(0,4N)$
 Dus $t = 3,32 \cdot \log(0,4N)$.
- b** $t = 1 + {}^2\log(0,2N)$
 $t = 1 + {}^2\log(0,2) + {}^2\log(N)$
 $t = 1 + {}^2\log(0,2) + \frac{{}^3\log(N)}{{}^3\log(2)}$
 $t \approx -1,32 + 1,58 \cdot {}^3\log(N)$
 Dus $t = -1,32 + 1,58 \cdot {}^3\log(N)$.
- c** $x = 64 \cdot 8^y$
 $x = 64 \cdot (3^{{}^3\log(8)})^y$
 $x \approx 64 \cdot (3^{1,893})^y$
 $x \approx 64 \cdot 3^{1,893y}$
 Dus $x = 64 \cdot 3^{1,893y}$.

41 a $N = 34 \cdot 3^{2t+3}$

$$34 \cdot 3^{2t+3} = N$$

$$3^{2t+3} = \frac{1}{34}N$$

$$2t + 3 = {}^3\log\left(\frac{1}{34}N\right)$$

$$2t = -3 + {}^3\log\left(\frac{1}{34}\right) + {}^3\log(N)$$

$$t = -1\frac{1}{2} + \frac{1}{2} \cdot {}^3\log\left(\frac{1}{34}\right) + \frac{1}{2} \cdot {}^3\log(N)$$

$$t \approx -3,10 + 0,5 \cdot {}^3\log(N)$$

$$\text{Dus } t = -3,10 + 0,5 \cdot {}^3\log(N).$$

b $y = 13 \cdot 2^{3x-1}$

$$13 \cdot 2^{3x-1} = y$$

$$2^{3x-1} = \frac{1}{13}y$$

$$3x - 1 = {}^2\log\left(\frac{1}{13}y\right)$$

$$3x = 1 + {}^2\log\left(\frac{1}{13}y\right)$$

$$3x = {}^2\log(2) + {}^2\log\left(\frac{1}{13}y\right)$$

$$3x = {}^2\log\left(\frac{2}{13}y\right)$$

$$x = \frac{1}{3} \cdot {}^2\log\left(\frac{2}{13}y\right)$$

$$x \approx 0,33 \cdot {}^2\log(0,15y)$$

$$\text{Dus } x = 0,33 \cdot {}^2\log(0,15y).$$

c $y = 3 \cdot \log(x) - 5$

$$3 \cdot \log(x) = y + 5$$

$$\log(x) = \frac{1}{3}y + \frac{5}{3}$$

$$x = 10^{\frac{1}{3}y + \frac{5}{3}} = 10^{\frac{1}{3}y} \cdot 10^{\frac{5}{3}} \approx 46,42 \cdot 10^{0,33y}$$

$$\text{Dus } x = 46,42 \cdot 10^{0,33y}.$$

d $K = 1 - \frac{1}{2} \cdot {}^3\log(2v)$

$$\frac{1}{2} \cdot {}^3\log(2v) = 1 - K$$

$${}^3\log(2v) = 2 - 2K$$

$$2v = 3^{2-2K}$$

$$v = \frac{1}{2} \cdot 3^{2-2K}$$

$$v = \frac{1}{2} \cdot 9 \cdot (3^{-2})^K$$

$$v = 4\frac{1}{2} \cdot \left(\frac{1}{9}\right)^K$$

$$\text{Dus } v = 4\frac{1}{2} \cdot \left(\frac{1}{9}\right)^K.$$

42 a ${}^2\log(4t - 1) = 5N + 3$

$$4t - 1 = 2^{5N+3}$$

$$4t = 1 + (2^5)^N \cdot 2^3$$

$$4t = 1 + 8 \cdot 32^N$$

$$4t = 1 + 8 \cdot (10^{\log(32)})^N$$

$$4t \approx 1 + 8 \cdot (10^{1,51})^N$$

$$t \approx \frac{1}{4} + 2 \cdot 10^{1,51N}$$

$$\text{Dus } t = \frac{1}{4} + 2 \cdot 10^{1,51N}.$$

b $M = 14 \cdot 1,3^{4N-3}$

$$14 \cdot 1,3^{4N-3} = M$$

$$1,3^{4N-3} = \frac{1}{14}M$$

$$4N - 3 = {}^{1,3}\log\left(\frac{1}{14}M\right)$$

$$4N = 3 + {}^{1,3}\log\left(\frac{1}{14}\right) + {}^{1,3}\log(M)$$

$$4N = 3 + {}^{1,3}\log\left(\frac{1}{14}\right) + \frac{\log(M)}{\log(1,3)}$$

$$N = \frac{3 + {}^{1,3}\log\left(\frac{1}{14}\right)}{4} + \frac{\log(M)}{4 \cdot \log(1,3)}$$

$$N \approx -1,76 + 2,19 \cdot \log(M)$$

$$\text{Dus } N = -1,76 + 2,19 \cdot \log(M).$$

c $w = 125 \cdot \log(3T + 15) - 8$

$$125 \cdot \log(3T + 15) = w + 8$$

$$\log(3T + 15) = 0,008w + 0,064$$

$$3T + 15 = 10^{0,008w + 0,064}$$

$$3T = -15 + (10^{0,008})^w \cdot 10^{0,064}$$

$$T \approx -5 + 0,386 \cdot 1,019^w$$

$$\text{Dus } T = -5 + 0,386 \cdot 1,019^w.$$

d $p = 10 \cdot 5^{0,1q-3}$

$$10 \cdot 5^{0,1q-3} = p$$

$$5^{0,1q-3} = 0,1p$$

$$0,1q - 3 = {}^5\log(0,1p)$$

$$0,1q = 3 + {}^5\log(0,1p)$$

$$0,1q = {}^5\log(125) + {}^5\log(0,1p)$$

$$0,1q = {}^5\log(12,5p)$$

$$0,1q = \frac{\log(12,5p)}{\log(5)}$$

$$q = \frac{10 \cdot \log(12,5p)}{\log(5)}$$

$$q \approx 14,31 \cdot \log(12,5p)$$

$$\text{Dus } q = 14,31 \cdot \log(12,5p).$$

43 a $f(x) = g_a(x)$ geeft ${}^2\log(x) = {}^2\log(x\sqrt{x}) + a$

$${}^2\log(x) = {}^2\log(x^{1\frac{1}{2}}) + a$$

$${}^2\log(x) = 1\frac{1}{2} \cdot {}^2\log(x) + a$$

$$\frac{1}{2} \cdot {}^2\log(x) = -a$$

$${}^2\log(x) = -2a$$

$$x = 2^{-2a} = (2^{-2})^a = \left(\frac{1}{4}\right)^a$$

$$\left. \begin{array}{l} f(x) = {}^2\log(x) \\ {}^2\log(x) = -2a \end{array} \right\} y = -2a$$

$$\text{Dus } S_a\left(\left(\frac{1}{4}\right)^a, -2a\right).$$

- b Uit $A_a B_a = 4$ en $y_{A_a} = 0$ volgt $y_{B_a} = 4 \vee y_{B_a} = -4$.

$$g_a(x) = 0 \text{ geeft } {}^2\log(x\sqrt{x}) + a = 0$$

$${}^2\log(x^{1\frac{1}{2}}) = -a$$

$$1\frac{1}{2} \cdot {}^2\log(x) = -a$$

$${}^2\log(x) = -\frac{2}{3}a$$

$$\left. \begin{aligned} f(x) &= {}^2\log(x) \\ {}^2\log(x) &= -\frac{2}{3}a \end{aligned} \right\} y_{B_a} = -\frac{2}{3}a$$

Dus $-\frac{2}{3}a = 4 \vee -\frac{2}{3}a = -4$ en dit geeft $a = -6 \vee a = 6$.

- c De grafiek van g_a^{inv} snijdt de y -as in $(0, 64)$, dus de grafiek van g_a snijdt de x -as in $(64, 0)$.

$$g_a(64) = 0 \text{ geeft } {}^2\log(64\sqrt{64}) + a = 0$$

$$a = -{}^2\log(64 \cdot 8)$$

$$a = -{}^2\log(512)$$

$$a = -{}^2\log(2^9)$$

$$a = -9$$

$$\text{Dus } g_{-9}(x) = {}^2\log(x\sqrt{x}) - 9.$$

Voor g_{-9} geldt $y = {}^2\log(x\sqrt{x}) - 9$, dus voor g_{-9}^{inv} geldt $x = {}^2\log(y\sqrt{y}) - 9$.

$$x = {}^2\log(y\sqrt{y}) - 9 \text{ geeft } {}^2\log(y\sqrt{y}) = x + 9$$

$${}^2\log(y^{1\frac{1}{2}}) = x + 9$$

$$1\frac{1}{2} \cdot {}^2\log(y) = x + 9$$

$${}^2\log(y) = \frac{2}{3}x + 6$$

$$y = 2^{\frac{2}{3}x + 6} = 2^{\frac{2}{3}x} \cdot 2^6 = 64 \cdot 2^{\frac{2}{3}x}$$

$$\text{Dus } g_{-9}^{\text{inv}}(x) = 64 \cdot 2^{\frac{2}{3}x}.$$

44 ${}^2\log(N) = -1,9 + 0,3 \cdot {}^3\log(t)$

$$0,3 \cdot {}^3\log(t) = 1,9 + {}^2\log(N)$$

$${}^3\log(t) = \frac{1,9}{0,3} + \frac{1}{0,3} \cdot {}^2\log(N)$$

$$t = 3^{\frac{1,9}{0,3} + \frac{1}{0,3} \cdot {}^2\log(N)}$$

$$t = 3^{\frac{1,9}{0,3}} \cdot 3^{\frac{1}{0,3} \cdot \frac{{}^2\log(N)}{{}^2\log(2)}}$$

$$t = 3^{\frac{1,9}{0,3}} \cdot (3^{{}^3\log(N)})^{\frac{1}{0,3 \cdot {}^2\log(2)}}$$

$$t = 3^{\frac{1,9}{0,3}} \cdot N^{\frac{1}{0,3 \cdot {}^2\log(2)}}$$

$$t \approx 1051 N^{5,28}$$

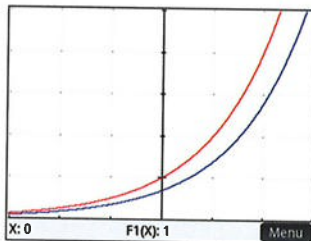
$$\text{Dus } t = 1051 N^{5,28}.$$

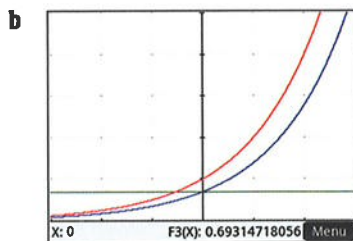
9.3 Het grondtal e

Bladzijde 28

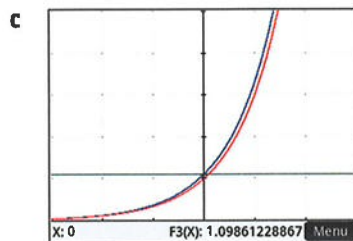
45

a





$$c \approx 0,6931$$



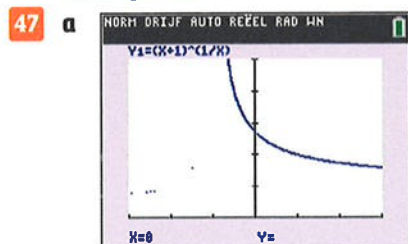
Dus ook hier is $\frac{y_2}{y_1}$ constant en wel ongeveer 1,0986.

46 a
$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h \rightarrow 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim_{h \rightarrow 0} \frac{2^x \cdot (2^h - 1)}{h} = \lim_{h \rightarrow 0} \frac{2^h - 1}{h} \cdot 2^x$$

b
$$f'(0) = \lim_{h \rightarrow 0} \frac{2^h - 1}{h} \cdot 2^0 = \lim_{h \rightarrow 0} \frac{2^h - 1}{h}$$

c
$$f'(x) = \lim_{h \rightarrow 0} \frac{2^h - 1}{h} \cdot 2^x = f'(0) \cdot 2^x$$

Bladzijde 29



b Omdat voor $x = 0$ de exponent $\frac{1}{x}$ niet bestaat.

c

x	y_1
0,01	2,7048
0,001	2,7169
0,0001	2,7181
0,00001	2,7183

d $g \approx 2,718$

Bladzijde 30

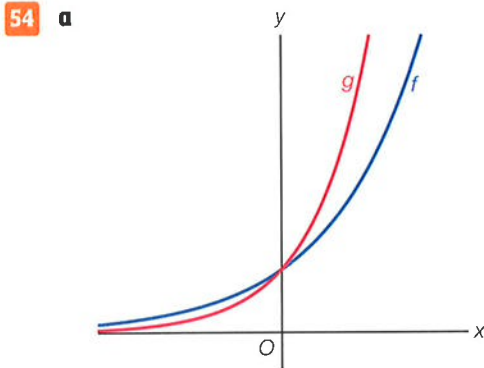
48 a Dit mag omdat $e^x \neq 0$ voor elke x , dus $e^x = 0$ geeft geen oplossing.

b $e^x = 1$ geeft $x = 0$ en $e^x = -3$ heeft geen oplossing.

Bladzijde 31

- 49**
- a** $2e^2 - e^2 = e^2$
- b** $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$
- c** $5e^2 \cdot 3e^3 = 15e^5$
- d** $\frac{12e^6}{4e^2} = 3e^4$
- e** $e^{5x} \cdot e^x = e^{6x}$
- f** $e^x \cdot e^2 = e^{x+2}$
- g** $5e^x - 3e^x = 2e^x$
- h** $e^x(e^2 + 1) = e^{x+2} + e^x$
- i** $e^x(e^x + 1) = e^{2x} + e^x$
- j** $(e^x + 1)^2 = (e^x)^2 + 2 \cdot e^x \cdot 1 + 1 = e^{2x} + 2e^x + 1$
- k** $(e^{3x} + 3)^2 = (e^{3x})^2 + 2 \cdot e^{3x} \cdot 3 + 9 = e^{6x} + 6e^{3x} + 9$
- l** $\frac{6e^{2x} - e^x}{e^x} = 6e^x - 1$
- 50**
- a** $(2 + 3e^{\frac{1}{2}x})^2 = 4 + 2 \cdot 2 \cdot 3e^{\frac{1}{2}x} + (3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$
- b** $(e^x + e^{-x})^2 = (e^x)^2 + 2 \cdot e^x \cdot e^{-x} + (e^{-x})^2 = e^{2x} + 2 + e^{-2x}$
- c** $\frac{e^{2x} - 4}{e^x + 2} = \frac{(e^x + 2)(e^x - 2)}{e^x + 2} = e^x - 2$
- 51**
- a** $(2x + 4)e^x = 0$
 $2x + 4 = 0$
 $2x = -4$
 $x = -2$
- b** $x^2 e^x = 3x e^x$
 $x^2 = 3x$
 $x = 0 \vee x = 3$
- c** $x^2 e^x = e^x$
 $x^2 = 1$
 $x = 1 \vee x = -1$
- d** $e^{3x} - e^x = 0$
 $e^{3x} = e^x$
 $3x = x$
 $x = 0$
- e** $e^{4x} - 1 = 0$
 $e^{4x} = 1$
 $4x = 0$
 $x = 0$
- f** $e^x \cdot e^x = e^6$
 $e^{2x} = e^6$
 $2x = 6$
 $x = 3$
- 52**
- a** $e^x + e^x = 2e^6$
 $2e^x = 2e^6$
 $e^x = e^6$
 $x = 6$
- b** $\frac{e^{5x}}{e^x} = e$
 $e^{4x} = e^1$
 $4x = 1$
 $x = \frac{1}{4}$
- c** $2x e^x + e^x = 0$
 $e^x(2x + 1) = 0$
 $2x + 1 = 0$
 $2x = -1$
 $x = -\frac{1}{2}$
- d** $e^{x+2} - \sqrt{e} = 0$
 $e^{x+2} = \sqrt{e}$
 $e^{x+2} = e^{\frac{1}{2}}$
 $x + 2 = \frac{1}{2}$
 $x = -1\frac{1}{2}$
- e** $e^{2x} + e^x = 2$
 $(e^x)^2 + e^x = 2$
 Stel $e^x = u$.
 $u^2 + u = 2$
 $u^2 + u - 2 = 0$
 $(u - 1)(u + 2) = 0$
 $u = 1 \vee u = -2$
 $e^x = 1 \vee e^x = -2$
 $x = 0$
- f** $e^{6x} + 1 = 2e^{3x}$
 $(e^{3x})^2 + 1 = 2e^{3x}$
 Stel $e^{3x} = u$.
 $u^2 + 1 = 2u$
 $u^2 - 2u + 1 = 0$
 $(u - 1)^2 = 0$
 $u = 1$
 $e^{3x} = 1$
 $3x = 0$
 $x = 0$

53 $e^{2x} + e^3 = (e + 1)e^{x+1}$
 $e^{2x} + e^3 = (e + 1)e^x \cdot e$
 $e^{2x} + e^3 = (e^2 + e)e^x$
 $(e^x)^2 - (e^2 + e)e^x + e^3 = 0$
 Stel $e^x = u$.
 $u^2 - (e^2 + e)u + e^3 = 0$
 $(u - e^2)(u - e) = 0$
 $u = e^2 \vee u = e$
 $e^x = e^2 \vee e^x = e$
 $x = 2 \vee x = 1$



- b $2 < e < 3$, dus voor $x > 0$ is $2^x < e^x < 3^x$ en voor $x < 0$ is $3^x < e^x < 2^x$.
 Voor $x = 0$ is $2^x = e^x = 3^x = 1$.
 Dus de grafiek van $h(x) = e^x$ ligt voor $x \neq 0$ tussen de grafieken van $f(x) = 2^x$ en $g(x) = 3^x$.

Bladzijde 32

- 55 De productregel is gebruikt in de onderdelen b en c.
 De quotiëntregel is gebruikt in het onderdeel d.
 De kettingregel is gebruikt in de onderdelen c en d.

56 a $f(x) = e^x + 2$ geeft $f'(x) = e^x$
 b $f(x) = 2e^x + \frac{1}{x} = 2e^x + x^{-1}$ geeft $f'(x) = 2e^x - x^{-2} = 2e^x - \frac{1}{x^2}$
 c $f(x) = xe^x + 4$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (x + 1)e^x$
 d $f(x) = \frac{x}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} = \frac{(1 - x)e^x}{(e^x)^2} = \frac{1 - x}{e^x}$
 e $f(x) = \frac{2e^x}{x - 1}$ geeft $f'(x) = \frac{(x - 1) \cdot 2e^x - 2e^x \cdot 1}{(x - 1)^2} = \frac{2xe^x - 2e^x - 2e^x}{(x - 1)^2} = \frac{(2x - 4)e^x}{(x - 1)^2}$
 f $f(x) = (2x - 4)e^x$ geeft $f'(x) = 2 \cdot e^x + (2x - 4) \cdot e^x = (2x - 2)e^x$

Bladzijde 33

57 a $f(x) = e^{x^2+x}$ geeft $f'(x) = e^{x^2+x} \cdot (2x + 1) = (2x + 1)e^{x^2+x}$
 b $f(x) = x^2 + 2e^{3x}$ geeft $f'(x) = 2x + 2 \cdot e^{3x} \cdot 3 = 2x + 6e^{3x}$
 c $f(x) = xe^{x^2}$ geeft $f'(x) = 1 \cdot e^{x^2} + x \cdot e^{x^2} \cdot 2x = (2x^2 + 1)e^{x^2}$
 d $f(x) = \frac{2e^{-x-1}}{x^2}$ geeft
 $f'(x) = \frac{x^2 \cdot 2e^{-x-1} \cdot -1 - 2e^{-x-1} \cdot 2x}{x^4} = \frac{-2x^2e^{-x-1} - 4xe^{-x-1}}{x^4} = \frac{-2x(x+2)e^{-x-1}}{x^4} = \frac{-2(x+2)e^{-x-1}}{x^3}$
 e $f(x) = 3xe^{2x-1}$ geeft $f'(x) = 3 \cdot e^{2x-1} + 3x \cdot e^{2x-1} \cdot 2 = (6x + 3)e^{2x-1}$
 f $f(x) = \frac{e^{2x}}{e^{2x} + 1}$ geeft $f'(x) = \frac{(e^{2x} + 1) \cdot e^{2x} \cdot 2 - e^{2x} \cdot e^{2x} \cdot 2}{(e^{2x} + 1)^2} = \frac{2e^{4x} + 2e^{2x} - 2e^{4x}}{(e^{2x} + 1)^2} = \frac{2e^{2x}}{(e^{2x} + 1)^2}$

58 a $-\frac{1}{e^2} \approx -0,135$ c $1\frac{1}{3}e^3 \approx 26,781$
 b $\frac{3e}{(e+2)^2} \approx 0,366$ d $\frac{e^2}{2e-3} \approx 3,033$

59 a $f(x) = xe^x + 2$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (1+x)e^x$
 b $f'(x) = 0$ geeft $(1+x)e^x = 0$
 $1+x = 0$
 $x = -1$

min. is $f(-1) = -e^{-1} + 2 = 2 - \frac{1}{e}$

c $f'(x) = (1+x)e^x$ geeft $f''(x) = 1 \cdot e^x + (1+x) \cdot e^x = (2+x)e^x$

d $f''(x) = 0$ geeft $(2+x)e^x = 0$
 $2+x = 0$
 $x = -2$

$f(-2) = -2e^{-2} + 2 = 2 - \frac{2}{e^2}$

Dus de coördinaten van het buigpunt zijn $\left(-2, 2 - \frac{2}{e^2}\right)$.

e Stel $k: y = ax + b$ met $a = f'(-2) = (1-2)e^{-2} = -\frac{1}{e^2}$.

$$\left. \begin{array}{l} y = -\frac{1}{e^2}x + b \\ \text{door } \left(-2, 2 - \frac{2}{e^2}\right) \end{array} \right\} \begin{array}{l} -\frac{1}{e^2} \cdot -2 + b = 2 - \frac{2}{e^2} \\ \frac{2}{e^2} + b = 2 - \frac{2}{e^2} \\ b = 2 - \frac{4}{e^2} \end{array}$$

Dus $k: y = -\frac{1}{e^2}x + 2 - \frac{4}{e^2}$.

60 a $f(x) = -xe^x$ geeft $f'(x) = -1 \cdot e^x + -x \cdot e^x = (-x-1)e^x$
 $f'(x) = 0$ geeft $(-x-1)e^x = 0$
 $-x-1 = 0$
 $-x = 1$
 $x = -1$

max. is $f(-1) = e^{-1} = \frac{1}{e}$

b $f'(x) = (-x-1)e^x$ geeft $f''(x) = -1 \cdot e^x + (-x-1) \cdot e^x = (-x-2)e^x$
 $f''(x) = 0$ geeft $(-x-2)e^x = 0$
 $-x-2 = 0$
 $-x = 2$
 $x = -2$

Stel $k: y = ax + b$ met $a = f'(-2) = (2-1)e^{-2} = \frac{1}{e^2}$.

$$\left. \begin{array}{l} y = \frac{1}{e^2}x + b \\ f(-2) = 2e^{-2} = \frac{2}{e^2} \end{array} \right\} \begin{array}{l} \frac{1}{e^2} \cdot -2 + b = \frac{2}{e^2} \\ \frac{-2}{e^2} + b = \frac{2}{e^2} \\ b = \frac{4}{e^2} \end{array}$$

Dus $k: y = \frac{1}{e^2}x + \frac{4}{e^2}$.

61 a $f(x) = 0$ geeft $(x^2 - 3)e^x = 0$

$$x^2 - 3 = 0$$

$$x^2 = 3$$

$$x = \sqrt{3} \vee x = -\sqrt{3}$$

De nulpunten zijn $\sqrt{3}$ en $-\sqrt{3}$.

b $f(x) = (x^2 - 3)e^x$ geeft $f'(x) = 2x \cdot e^x + (x^2 - 3) \cdot e^x = (x^2 + 2x - 3)e^x$

$$f'(x) = 0 \text{ geeft } (x^2 + 2x - 3)e^x = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x - 1)(x + 3) = 0$$

$$x = 1 \vee x = -3$$

$$\text{max. is } f(-3) = (9 - 3)e^{-3} = \frac{6}{e^3} \text{ en min. is } f(1) = (1 - 3)e^1 = -2e.$$

c $f'(x) = (x^2 + 2x - 3)e^x$ geeft $f''(x) = (2x + 2) \cdot e^x + (x^2 + 2x - 3) \cdot e^x = (x^2 + 4x - 1)e^x$

$$f''(x) = 0 \text{ geeft } (x^2 + 4x - 1)e^x = 0$$

$$x^2 + 4x - 1 = 0$$

$$D = 4^2 - 4 \cdot 1 \cdot (-1) = 20, \text{ dus } \sqrt{D} = \sqrt{20} = 2\sqrt{5}$$

$$x = \frac{-4 + 2\sqrt{5}}{2} \vee x = \frac{-4 - 2\sqrt{5}}{2}$$

$$x = -2 + \sqrt{5} \vee x = -2 - \sqrt{5}$$

De x -coördinaten van de buigpunten zijn $-2 + \sqrt{5}$ en $-2 - \sqrt{5}$.

Bladzijde 34

62 a $f(x) = \frac{1}{2}e^{2x}$ geeft $f'(x) = \frac{1}{2} \cdot e^{2x} \cdot 2 = e^{2x}$

Stel $k: y = ax + b$ met $a = f'(-1) = e^{-2} = \frac{1}{e^2}$.

$$\left. \begin{array}{l} y = \frac{1}{e^2}x + b \\ f(-1) = \frac{1}{2e^2}, \text{ dus } A\left(-1, \frac{1}{2e^2}\right) \end{array} \right\} \begin{array}{l} \frac{1}{e^2} \cdot (-1) + b = \frac{1}{2e^2} \\ -\frac{1}{e^2} + b = \frac{1}{2e^2} \\ b = \frac{3}{2e^2} \end{array}$$

Dus $k: y = \frac{1}{e^2}x + \frac{3}{2e^2}$.

$$g(x) = \frac{1}{e^{x+3}} = e^{-x-3} \text{ geeft } g'(x) = e^{-x-3} \cdot (-1) = -e^{-x-3}$$

Stel $l: y = ax + b$ met $a = g'(-1) = -e^{1-3} = -e^{-2} = -\frac{1}{e^2}$.

$$\left. \begin{array}{l} y = -\frac{1}{e^2}x + b \\ g(-1) = e^{1-3} = \frac{1}{e^2}, \text{ dus } B\left(-1, \frac{1}{e^2}\right) \end{array} \right\} \begin{array}{l} -\frac{1}{e^2} \cdot (-1) + b = \frac{1}{e^2} \\ \frac{1}{e^2} + b = \frac{1}{e^2} \\ b = 0 \end{array}$$

Dus $l: y = -\frac{1}{e^2}x$.

k en l snijden geeft $\frac{1}{e^2}x + \frac{3}{2e^2} = -\frac{1}{e^2}x$

$$\frac{2}{e^2}x = -\frac{3}{2e^2}$$

$$x = -\frac{3}{2e^2} \cdot \frac{e^2}{2}$$

$$x = -\frac{3}{4}$$

b $h(x) = f(x) + g(x) = \frac{1}{2}e^{2x} + \frac{1}{e^{x+3}}$ geeft $h'(x) = e^{2x} - e^{-x-3}$

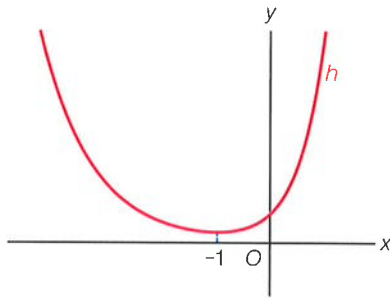
$h'(x) = 0$ geeft $e^{2x} - e^{-x-3} = 0$

$$e^{2x} = e^{-x-3}$$

$$2x = -x - 3$$

$$3x = -3$$

$$x = -1$$



min. is $h(-1) = \frac{1}{2}e^{-2} + \frac{1}{e^2} = \frac{1}{2e^2} + \frac{2}{2e^2} = \frac{3}{2e^2}$

Het bereik is dus $B_h = \left[\frac{3}{2e^2}, \rightarrow \right)$.

63 a $h(4) = 5$ geeft $2(e^{\frac{1}{2} \cdot 4} + e^{-\frac{1}{2} \cdot 4}) + c = 5$

$$2e^1 + 2e^{-1} + c = 5$$

$$c = 5 - 2e - \frac{2}{e}$$

b $h(x) = 5(e^{0,1x} + e^{-0,1x}) - 5,81$ geeft $h'(x) = 5(e^{0,1x} \cdot 0,1 + e^{-0,1x} \cdot -0,1) = 0,5(e^{0,1x} - e^{-0,1x})$

De helling in B is $h'(4) = 0,5(e^{0,1 \cdot 4} - e^{-0,1 \cdot 4}) = 0,5(e^{0,4} - e^{-0,4}) \approx 0,41$.

c $h_k(0) = 0$ geeft $\frac{1}{2k}(e^0 + e^0 - e^{4k} - e^{-4k}) + 5 = 0$ oftewel $\frac{1}{2k}(2 - e^{4k} - e^{-4k}) + 5 = 0$.

Voer in $y_1 = \frac{1}{2x}(2 - e^{4x} - e^{-4x}) + 5$.

De optie nulpunt geeft $x = 0,469...$

Dus $k \approx 0,47$.

Bladzijde 35

64 a $f(x) = 2xe^{-x^2}$ geeft $f'(x) = 2 \cdot e^{-x^2} + 2x \cdot e^{-x^2} \cdot -2x = (2 - 4x^2)e^{-x^2}$

$f'(x) = 0$ geeft $(2 - 4x^2)e^{-x^2} = 0$

$$2 - 4x^2 = 0$$

$$4x^2 = 2$$

$$x^2 = \frac{1}{2}$$

$$x = \sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} \vee x = -\frac{1}{2}\sqrt{2}$$

$$y_A = 2 \cdot -\frac{1}{2}\sqrt{2} \cdot e^{-\frac{1}{2}} = -\frac{\sqrt{2}}{e^{\frac{1}{2}}} = -\frac{\sqrt{2}}{\sqrt{e}} = -\sqrt{\frac{2}{e}}$$

$$y_B = 2 \cdot \frac{1}{2}\sqrt{2} \cdot e^{\frac{1}{2}} = \sqrt{\frac{2}{e}}$$

$$y_B - y_A = \sqrt{\frac{2}{e}} - -\sqrt{\frac{2}{e}} = 2\sqrt{\frac{2}{e}}$$

Dus $a = \frac{2}{e}$.

b $f'(x) = (2 - 4x^2)e^{-x^2}$ geeft
 $f''(x) = -8x \cdot e^{-x^2} + (2 - 4x^2) \cdot e^{-x^2} \cdot -2x = -8x \cdot e^{-x^2} + (8x^3 - 4x) \cdot e^{-x^2} = (8x^3 - 12x)e^{-x^2}$
 $f''(x) = 0$ geeft $(8x^3 - 12x)e^{-x^2} = 0$
 $8x^3 - 12x = 0$
 $4x(2x^2 - 3) = 0$
 $4x = 0 \vee 2x^2 = 3$
 $x = 0 \vee x^2 = 1\frac{1}{2}$
 $x = 0 \vee x = \sqrt{1\frac{1}{2}} = \frac{1}{2}\sqrt{6} \vee x = -\sqrt{1\frac{1}{2}} = -\frac{1}{2}\sqrt{6}$

$$f(0) = 0$$

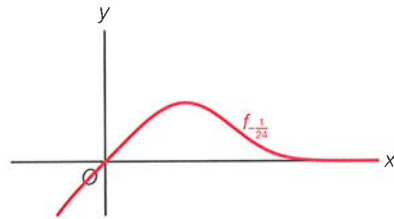
$$f\left(\frac{1}{2}\sqrt{6}\right) = \sqrt{6} \cdot e^{-\frac{1}{2}} = \frac{\sqrt{6}}{e\sqrt{e}}$$

$$f\left(-\frac{1}{2}\sqrt{6}\right) = -\sqrt{6} \cdot e^{-\frac{1}{2}} = -\frac{\sqrt{6}}{e\sqrt{e}}$$

De coördinaten van de buigpunten van de grafiek van f zijn $(0, 0)$,

$$\left(\frac{1}{2}\sqrt{6}, \frac{\sqrt{6}}{e\sqrt{e}}\right) \text{ en } \left(-\frac{1}{2}\sqrt{6}, -\frac{\sqrt{6}}{e\sqrt{e}}\right).$$

65 $f_a(x) = x e^{ax^3}$ geeft $f'_a(x) = 1 \cdot e^{ax^3} + x \cdot e^{ax^3} \cdot 3ax^2 = (1 + 3ax^3)e^{ax^3}$
 $f'_a(2) = 0$ geeft $(1 + 3a \cdot 2^3)e^{a \cdot 2^3} = 0$
 $(1 + 3a \cdot 8)e^{8a} = 0$
 $(1 + 24a)e^{8a} = 0$
 $1 + 24a = 0$
 $24a = -1$
 $a = -\frac{1}{24}$
 $a = -\frac{1}{24}$ geeft $f_{-\frac{1}{24}}(x) = x e^{-\frac{1}{24}x^3}$



De extreme waarde voor $x = 2$ is een maximum.

9.4 De natuurlijke logaritme

Bladzijde 37

66 a $f(x) = 2^x$ geeft $f^{\text{inv}}(x) = {}^2\log(x)$
b $g(x) = e^x$ geeft $g^{\text{inv}}(x) = {}^e\log(x)$

Bladzijde 38

67 a $\ln(e) = 1$
b $\ln(e\sqrt{e}) = \ln(e \cdot e^{\frac{1}{2}}) = \ln(e^{\frac{3}{2}}) = 1\frac{1}{2}$
c $\ln\left(\frac{1}{e}\right) = \ln(e^{-1}) = -1$
d $\ln(1) = 0$
e $3 \ln(e \cdot \sqrt[3]{e}) = 3 \ln(e \cdot e^{\frac{1}{3}}) = 3 \ln(e^{\frac{4}{3}}) = 3 \cdot 1\frac{1}{3} = 4$
f $\ln^2(e^3) = 3^2 = 9$
g $\ln^3(e^2) = 2^3 = 8$
h $e^{\ln(7)} + e^{2\ln(7)} = 7 + e^{\ln(7^2)} = 7 + 7^2 = 56$
i $e^{\frac{1}{2}\ln(5)} = e^{\ln(5^{\frac{1}{2}})} = 5^{\frac{1}{2}} = \sqrt{5}$
j $e^{\ln(10)} \cdot e^{\ln(3)} = 10 \cdot 3 = 30$

68 a $e^{3x} = 12$
 $3x = \ln(12)$
 $x = \frac{1}{3} \ln(12) \approx 0,828$

b $5e^{2x} = 60$
 $e^{2x} = 12$
 $2x = \ln(12)$
 $x = \frac{1}{2} \ln(12) \approx 1,242$

c $6 + e^{0,5x+2} = 10$
 $e^{0,5x+2} = 4$
 $0,5x + 2 = \ln(4)$
 $x + 4 = 2 \ln(4)$
 $x = 2 \ln(4) - 4 \approx -1,227$

d $\frac{3}{5e^{2x-1}} = 10$
 $50e^{2x-1} = 3$
 $e^{2x-1} = \frac{3}{50}$
 $2x - 1 = \ln\left(\frac{3}{50}\right)$
 $2x = 1 + \ln\left(\frac{3}{50}\right)$
 $x = \frac{1}{2} + \frac{1}{2} \ln\left(\frac{3}{50}\right) \approx -0,907$

69 a $2 \ln(3) + \ln(4) = \ln(3^2) + \ln(4) = \ln(9) + \ln(4) = \ln(9 \cdot 4) = \ln(36)$

b $\ln(20) - 3 \ln(2) = \ln(20) - \ln(2^3) = \ln(20) - \ln(8) = \ln\left(\frac{20}{8}\right) = \ln\left(2\frac{1}{2}\right)$

c $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$

d $1 + \ln(10) = \ln(e) + \ln(10) = \ln(10e)$

e $\frac{1}{2} + 2 \ln(6) = \ln(e^{\frac{1}{2}}) + \ln(6^2) = \ln(\sqrt{e}) + \ln(36) = \ln(36\sqrt{e})$

f $e + \ln(2) = \ln(e^e) + \ln(2) = \ln(2e^e)$

70 a $\ln(x) = -1$
 $x = e^{-1} = \frac{1}{e}$

b $4 \ln(x) = 2$
 $\ln(x) = \frac{1}{2}$
 $x = e^{\frac{1}{2}} = \sqrt{e}$

c $\ln(3x) = 3$
 $3x = e^3$
 $x = \frac{1}{3}e^3$

d $\ln(-x + 2) = -2$
 $-x + 2 = e^{-2}$
 $-x = -2 + \frac{1}{e^2}$
 $x = 2 - \frac{1}{e^2}$

e $\ln^2(x) = \frac{1}{4}$
 $\ln(x) = \frac{1}{2} \vee \ln(x) = -\frac{1}{2}$
 $x = e^{\frac{1}{2}} = \sqrt{e} \vee x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

f $\ln(x) = 1 + \ln(5)$
 $\ln(x) = \ln(e) + \ln(5)$
 $\ln(x) = \ln(5e)$
 $x = 5e$

Bladzijde 39

71 a $3x \ln(x) = 2 \ln(x)$
 $\ln(x) = 0 \vee 3x = 2$
 $x = 1 \vee x = \frac{2}{3}$

b $\ln^2(x) - \ln(x) = 0$
 $\ln(x)(\ln(x) - 1) = 0$
 $\ln(x) = 0 \vee \ln(x) = 1$
 $x = 1 \vee x = e$

c $x^2 \ln(x+1) = 4 \ln(x+1)$
 $\ln(x+1) = 0 \vee x^2 = 4$
 $x+1 = 1 \vee x = 2 \vee x = -2$
 $x = 0 \vee x = 2 \vee x = -2$
vold. niet

d $4e^{1-3x} = 20$
 $e^{1-3x} = 5$
 $1 - 3x = \ln(5)$
 $-3x = -1 + \ln(5)$
 $x = \frac{1}{3} - \frac{1}{3} \ln(5)$

e $\ln^2(x) - 2 \ln(x) - 3 = 0$
Stel $\ln(x) = u$.
 $u^2 - 2u - 3 = 0$
 $(u+1)(u-3) = 0$
 $u = -1 \vee u = 3$
 $\ln(x) = -1 \vee \ln(x) = 3$
 $x = e^{-1} = \frac{1}{e} \vee x = e^3$

f $\ln(x+3) - \ln(x-1) = \ln(2)$
 $\ln\left(\frac{x+3}{x-1}\right) = \ln(2)$
 $\frac{x+3}{x-1} = 2$
 $2x - 2 = x + 3$
 $x = 5$

$$\begin{aligned}
 \text{g} \quad & 2 \ln(x) = \ln(2) + \ln(x+4) \\
 & \ln(x^2) = \ln(2(x+4)) \\
 & x^2 = 2x + 8 \\
 & x^2 - 2x - 8 = 0 \\
 & (x+2)(x-4) = 0 \\
 & x = -2 \vee x = 4 \\
 & \text{vold. niet}
 \end{aligned}$$

$$\begin{aligned}
 \text{h} \quad & e^{x^2} = 100 \\
 & x^2 = \ln(100) \\
 & x = \sqrt{\ln(100)} \vee x = -\sqrt{\ln(100)}
 \end{aligned}$$

$$\begin{aligned}
 72 \quad \text{a} \quad & G = 100 \text{ geeft } F = 16(0,6 + \ln(100)) \approx 83 \\
 & \text{De hartslagfrequentie is 83 slagen per minuut.}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad & F = 78 \text{ geeft } 16(0,6 + \ln(G)) = 78 \\
 & 0,6 + \ln(G) = 4,875 \\
 & \ln(G) = 4,275 \\
 & G = e^{4,275} \approx 72
 \end{aligned}$$

Dus het hiermee corresponderende gewicht is 72 kg.

$$\begin{aligned}
 \text{c} \quad & F = 16(0,6 + \ln(G)) \\
 & 16(0,6 + \ln(G)) = F
 \end{aligned}$$

$$0,6 + \ln(G) = \frac{1}{16}F$$

$$\ln(G) = \frac{1}{16}F - 0,6$$

$$G = e^{\frac{1}{16}F - 0,6}$$

$$G = e^{\frac{1}{16}F} \cdot e^{-0,6}$$

$$G = e^{-0,6} \cdot (e^{\frac{1}{16}})^F$$

$$G \approx 0,549 \cdot 1,064^F$$

$$\text{Dus } G = 0,549 \cdot 1,064^F.$$

$$73 \quad \text{a} \quad 2^x = (e^{\ln(2)})^x = e^{\ln(2) \cdot x}$$

$$\text{b} \quad f(x) = 2^x = e^{\ln(2) \cdot x} \text{ geeft } f'(x) = e^{\ln(2) \cdot x} \cdot \ln(2) = 2^x \cdot \ln(2)$$

$$\begin{aligned}
 74 \quad \text{a} \quad & e^{\ln(x)} = x \text{ geeft } [e^{\ln(x)}]' = x' \\
 & e^{\ln(x)} \cdot [\ln(x)]' = 1 \\
 & e^{\ln(x)} \cdot f'(x) = 1
 \end{aligned}$$

$$\text{b} \quad e^{\ln(x)} \cdot f'(x) = 1$$

$$x \cdot f'(x) = 1$$

$$f'(x) = \frac{1}{x}$$

$$\text{c} \quad h(x) = {}^2\log(x) = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)} \cdot \ln(x) \text{ geeft } h'(x) = \frac{1}{\ln(2)} \cdot \frac{1}{x} = \frac{1}{x \ln(2)}$$

Bladzijde 40

$$75 \quad \text{a} \quad f(x) = \ln(ax) \text{ geeft } f'(x) = \frac{1}{ax} \cdot a = \frac{1}{x}$$

$$\text{b} \quad f(x) = \ln(2x) \text{ geeft } f'(x) = \frac{1}{x}$$

$$g(x) = \ln(x\sqrt{2}) \text{ geeft } g'(x) = \frac{1}{x}$$

$$\text{c} \quad f(x) = \ln(x^n) = n \cdot \ln(x) \text{ geeft } f'(x) = n \cdot \frac{1}{x} = \frac{n}{x}$$

$$\text{d} \quad f(x) = \ln(x^2) \text{ geeft } f'(x) = \frac{2}{x}$$

$$g(x) = \ln\left(\frac{1}{x}\right) \text{ geeft } g'(x) = \frac{-1}{x}$$

$$h(x) = \ln(\sqrt{x}) \text{ geeft } h'(x) = \frac{1}{2x}$$

Bladzijde 41

76 a $f(x) = 3^{4x-2}$ geeft $f'(x) = 3^{4x-2} \cdot \ln(3) \cdot 4 = 4 \cdot 3^{4x-2} \cdot \ln(3)$

b $f(x) = \frac{1 - \ln(x)}{x}$ geeft $f'(x) = \frac{x \cdot -\frac{1}{x} - (1 - \ln(x)) \cdot 1}{x^2} = \frac{-1 - 1 + \ln(x)}{x^2} = \frac{\ln(x) - 2}{x^2}$

c $f(x) = {}^2\log(4x - 1)$ geeft $f'(x) = \frac{1}{(4x - 1)\ln(2)} \cdot 4 = \frac{4}{(4x - 1)\ln(2)}$

d $f(x) = \frac{\ln(3x)}{x}$ geeft $f'(x) = \frac{x \cdot \frac{1}{x} - \ln(3x) \cdot 1}{x^2} = \frac{1 - \ln(3x)}{x^2}$

e $f(x) = x \ln(x^3)$ geeft $f'(x) = 1 \cdot \ln(x^3) + x \cdot \frac{3}{x} = \ln(x^3) + 3$

f $f(x) = \ln(x^2 + x)$ geeft $f'(x) = \frac{1}{x^2 + x} \cdot (2x + 1) = \frac{2x + 1}{x^2 + x}$

77 a $f(x) = \ln(2^x) = x \ln(2)$ geeft $f'(x) = \ln(2)$

b $f(x) = {}^2\log(x^2 + 1)$ geeft $f'(x) = \frac{1}{(x^2 + 1)\ln(2)} \cdot 2x = \frac{2x}{(x^2 + 1)\ln(2)}$

c $f(x) = x \ln^2(x)$ geeft $f'(x) = 1 \cdot \ln^2(x) + x \cdot 2 \ln(x) \cdot \frac{1}{x} = \ln^2(x) + 2 \ln(x)$

d $f(x) = x^2 \cdot {}^3\log(4x)$ geeft $f'(x) = 2x \cdot {}^3\log(4x) + x^2 \cdot \frac{1}{x \ln(3)} = 2x \cdot {}^3\log(4x) + \frac{x}{\ln(3)}$

e $f(x) = (2^x - 1) \cdot 2^x = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = 2^x \cdot \ln(2) \cdot (2 \cdot 2^x - 1)$

f $f(x) = \ln^2(4x)$ geeft $f'(x) = 2 \ln(4x) \cdot \frac{1}{x} = \frac{2 \ln(4x)}{x}$

78 a $x^n = (e^{\ln(x)})^n = e^{n \ln(x)}$

b $f(x) = e^{n \ln(x)}$ geeft $f'(x) = e^{n \ln(x)} \cdot \frac{n}{x}$

c $e^{n \ln(x)} \cdot \frac{n}{x} = x^n \cdot \frac{n}{x} = n \cdot \frac{x^n}{x} = nx^{n-1}$

Er is geen gebruikgemaakt van enige beperking van n , dus de regel geldt ook voor elke niet-gehele n van $\mathbb{R}$.

79 a $x - 2 > 0$ oftewel $x > 2$, dus $D_f = \langle 2, \rightarrow \rangle$.

$7 - x > 0$ oftewel $x < 7$, dus $D_g = \langle \leftarrow, 7 \rangle$.

$f(x) = g(x)$ geeft $\ln(x - 2) = \ln(7 - x)$

$$x - 2 = 7 - x$$

$$2x = 9$$

$$x = 4\frac{1}{2}$$

$f(x) \leq g(x)$ geeft $2 < x \leq 4\frac{1}{2}$

b $s(x) = f(x) + g(x) = \ln(x-2) + \ln(7-x)$ geeft $s'(x) = \frac{1}{x-2} + \frac{1}{7-x} \cdot -1 = \frac{1}{x-2} - \frac{1}{7-x}$

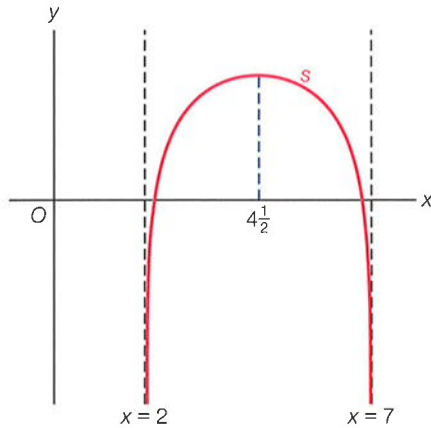
$$s'(x) = 0 \text{ geeft } \frac{1}{x-2} - \frac{1}{7-x} = 0$$

$$\frac{1}{x-2} = \frac{1}{7-x}$$

$$x-2 = 7-x$$

$$2x = 9$$

$$x = 4\frac{1}{2}$$



max. is $s(4\frac{1}{2}) = \ln(2\frac{1}{2}) + \ln(2\frac{1}{2}) = 2\ln(2\frac{1}{2})$

80 a $f(x) = \frac{x}{\ln(x)}$ geeft $f'(x) = \frac{\ln(x) \cdot 1 - x \cdot \frac{1}{x}}{\ln^2(x)} = \frac{\ln(x) - 1}{\ln^2(x)}$

Stel $k: y = ax + b$ met $a = f'\left(\frac{1}{e}\right) = \frac{\ln\left(\frac{1}{e}\right) - 1}{\ln^2\left(\frac{1}{e}\right)} = \frac{-1 - 1}{(-1)^2} = -2$.

$$\left. \begin{array}{l} y = -2x + b \\ f\left(\frac{1}{e}\right) = \frac{\frac{1}{e}}{\ln\left(\frac{1}{e}\right)} = \frac{\frac{1}{e}}{-1} = -\frac{1}{e}, \text{ dus } A\left(\frac{1}{e}, -\frac{1}{e}\right) \end{array} \right\} \begin{array}{l} -2 \cdot \frac{1}{e} + b = -\frac{1}{e} \\ b = \frac{1}{e} \end{array}$$

Dus $k: y = -2x + \frac{1}{e}$.

b $f'(x) = -6$ geeft $\frac{\ln(x) - 1}{\ln^2(x)} = -6$

$$\ln(x) - 1 = -6\ln^2(x)$$

Stel $\ln(x) = u$.

$$u - 1 = -6u^2$$

$$6u^2 + u - 1 = 0$$

$$D = 1^2 - 4 \cdot 6 \cdot -1 = 25$$

$$u = \frac{-1 \pm 5}{12} = \frac{1}{3} \vee u = \frac{-1 - 5}{12} = -\frac{1}{2}$$

$$\ln(x) = \frac{1}{3} \vee \ln(x) = -\frac{1}{2}$$

$$x = e^{\frac{1}{3}} \vee x = e^{-\frac{1}{2}}$$

$$f(e^{\frac{1}{3}}) = \frac{e^{\frac{1}{3}}}{\ln(e^{\frac{1}{3}})} = \frac{e^{\frac{1}{3}}}{\frac{1}{3}} = 3e^{\frac{1}{3}}, \text{ dus raakpunt}(e^{\frac{1}{3}}, 3e^{\frac{1}{3}}) = (\sqrt[3]{e}, 3 \cdot \sqrt[3]{e}).$$

$$f(e^{-\frac{1}{2}}) = \frac{e^{-\frac{1}{2}}}{\ln(e^{-\frac{1}{2}})} = \frac{e^{-\frac{1}{2}}}{-\frac{1}{2}} = -2e^{-\frac{1}{2}}, \text{ dus raakpunt}(e^{-\frac{1}{2}}, -2e^{-\frac{1}{2}}) = \left(\frac{1}{\sqrt{e}}, -\frac{2}{\sqrt{e}}\right).$$

Bladzijde 42

81 a $f(x) = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = (2^{2x+1} - 2^x) \ln(2) = (2^{2x+1} - 2^x) \ln(2)$
 $f'(x) = 0$ geeft $(2^{2x+1} - 2^x) \ln(2) = 0$
 $2^{2x+1} - 2^x = 0$
 $2^{2x+1} = 2^x$
 $2x + 1 = x$
 $x = -1$

min. is $f(-1) = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, dus $B_f = [-\frac{1}{4}, \rightarrow)$.

b $f'(x) = (2^{2x+1} - 2^x) \ln(2)$ geeft $f''(x) = (2^{2x+1} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2)) \ln(2) = (2^{2x+2} - 2^x) \ln^2(2)$
 $f''(x) = 0$ geeft $(2^{2x+2} - 2^x) \ln^2(2) = 0$
 $2^{2x+2} - 2^x = 0$
 $2^{2x+2} = 2^x$
 $2x + 2 = x$
 $x = -2$
 $f(-2) = 2^{-4} - 2^{-2} = \frac{1}{16} - \frac{1}{4} = -\frac{3}{16}$, dus buigpunt $(-2, -\frac{3}{16})$.

82 a $f(x) = \sqrt{x} \ln(\sqrt{x})$ geeft $f'(x) = \frac{1}{2\sqrt{x}} \cdot \ln(\sqrt{x}) + \sqrt{x} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{\ln(\sqrt{x}) + 1}{2\sqrt{x}}$

$$f'(x) = 0 \text{ geeft } \frac{\ln(\sqrt{x}) + 1}{2\sqrt{x}} = 0$$

$$\ln(\sqrt{x}) + 1 = 0$$

$$\ln(\sqrt{x}) = -1$$

$$\sqrt{x} = e^{-1}$$

$$x = e^{-2} = \frac{1}{e^2}$$

$$f\left(\frac{1}{e^2}\right) = e^{-1} \cdot \ln(e^{-1}) = \frac{1}{e} \cdot -1 = -\frac{1}{e}, \text{ dus } A\left(\frac{1}{e^2}, -\frac{1}{e}\right).$$

b $f'(x) = \frac{\ln(\sqrt{x}) + 1}{2\sqrt{x}}$ geeft

$$f''(x) = \frac{2\sqrt{x} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} - (\ln(\sqrt{x}) + 1) \cdot \frac{2}{2\sqrt{x}}}{(2\sqrt{x})^2} = \frac{\frac{1}{\sqrt{x}} - \frac{(\ln(\sqrt{x}) + 1)}{\sqrt{x}}}{4x} = \frac{\frac{\sqrt{x} - (\ln(\sqrt{x}) + 1)}{\sqrt{x}}}{4x\sqrt{x}} = \frac{-\ln(\sqrt{x})}{4x\sqrt{x}}$$

$$f''(x) = 0 \text{ geeft } -\ln(\sqrt{x}) = 0$$

$$\ln(\sqrt{x}) = 0$$

$$\sqrt{x} = 1$$

$$x = 1$$

$$\text{Stel } k: y = ax + b \text{ met } a = f'(1) = \frac{\ln(\sqrt{1}) + 1}{2\sqrt{1}} = \frac{0 + 1}{2} = \frac{1}{2}.$$

$$\left. \begin{array}{l} y = \frac{1}{2}x + b \\ f(1) = \sqrt{1} \ln(\sqrt{1}) = 0 \end{array} \right\} \frac{1}{2} \cdot 1 + b = 0$$

$$\text{Dus } k: y = \frac{1}{2}x - \frac{1}{2}.$$

83 a Op $t = 0$ is $T = 80$, dus $20 + pe^0 = 80$
 $20 + p \cdot 1 = 80$
 $p = 60$

b $T = 45$ geeft $20 + 60e^{-\frac{1}{60}t} = 45$

$$60e^{-\frac{1}{60}t} = 25$$

$$e^{-\frac{1}{60}t} = \frac{25}{60}$$

$$-\frac{1}{60}t = \ln\left(\frac{25}{60}\right)$$

$$t = -6 \ln\left(\frac{25}{60}\right) = 5,252\dots$$

Dus na 5 minuten en $0,252\dots \cdot 60 \approx 15$ seconden is de temperatuur van de koffie 45°C .

c $T = 20 + 60e^{-\frac{1}{6}t}$ geeft $\frac{dT}{dt} = 60e^{-\frac{1}{6}t} \cdot -\frac{1}{6} = -10e^{-\frac{1}{6}t}$

$t = 2$ geeft $\frac{dT}{dt} = -10e^{-\frac{1}{3}} \approx -7,2$

Twee minuten nadat de koffie is ingeschonken neemt de temperatuur van de koffie af met een snelheid van 7,2 °C per minuut.

d $T = 20 + 60e^{-\frac{1}{6}t}$

$60e^{-\frac{1}{6}t} = T - 20$

$e^{-\frac{1}{6}t} = \frac{1}{60}T - \frac{1}{3}$

$-\frac{1}{6}t = \ln\left(\frac{1}{60}T - \frac{1}{3}\right)$

$t = -6\ln\left(\frac{1}{60}T - \frac{1}{3}\right)$

Dus $a = -6$, $b = \frac{1}{60}$ en $c = -\frac{1}{3}$.

84 a $f_2(x) = \ln\left(\frac{2x}{2x+1}\right)$ geeft

$$f_2'(x) = \frac{1}{\left(\frac{2x}{2x+1}\right)} \cdot \frac{(2x+1) \cdot 2 - 2x \cdot 2}{(2x+1)^2} = \frac{2x+1}{2x} \cdot \frac{4x+2-4x}{(2x+1)^2} = \frac{2(2x+1)}{2x(2x+1)^2} = \frac{1}{x(2x+1)}$$

$rc_k = \frac{1}{3}$, dus $f_2'(x) = \frac{1}{3}$

$$\frac{1}{x(2x+1)} = \frac{1}{3}$$

$$x(2x+1) = 3$$

$$2x^2 + x - 3 = 0$$

$$D = 1^2 - 4 \cdot 2 \cdot -3 = 25$$

$$x = \frac{-1 \pm 5}{4} = 1 \vee x = \frac{-1 - 5}{4} = -1\frac{1}{2}$$

vold. niet

$f_2(1) = \ln\left(\frac{2}{2+1}\right) = \ln\left(\frac{2}{3}\right)$, dus $A(1, \ln(\frac{2}{3}))$.

b Voor f_a geldt $y = \ln\left(\frac{ax}{2x+1}\right)$, dus voor f_a^{inv} geldt $x = \ln\left(\frac{ay}{2y+1}\right)$.

$x = \ln\left(\frac{ay}{2y+1}\right)$ geeft $\ln\left(\frac{ay}{2y+1}\right) = x$

$$\frac{ay}{2y+1} = e^x$$

$$ay = (2y+1)e^x$$

$$ay = 2ye^x + e^x$$

$$ay - 2ye^x = e^x$$

$$y(a - 2e^x) = e^x$$

$$y = \frac{e^x}{a - 2e^x}$$

Dus $f_a^{\text{inv}}(x) = \frac{e^x}{a - 2e^x}$.

Dus voor $a = 3$ is $g(x) = \frac{e^x}{3 - 2e^x}$ de inverse van f_a .

Bladzijde 43

85 a $f(x) = 1$ geeft $e^{\ln^2(x) + 2\ln(x) - 3} = 1$

$$\ln^2(x) + 2\ln(x) - 3 = 0$$

Stel $\ln(x) = u$.

$$u^2 + 2u - 3 = 0$$

$$(u-1)(u+3) = 0$$

$$u = 1 \vee u = -3$$

$$\ln(x) = 1 \vee \ln(x) = -3$$

$$x = e \vee x = e^{-3} = \frac{1}{e^3}$$

$f(x) \leq 1$ geeft $\frac{1}{e^3} \leq x \leq e$

b $f(x) = e^{\ln^2(x) + 2 \ln(x) - 3}$ geeft

$$f'(x) = e^{\ln^2(x) + 2 \ln(x) - 3} \cdot \left(2 \ln(x) \cdot \frac{1}{x} + \frac{2}{x} \right) = e^{\ln^2(x) + 2 \ln(x) - 3} \cdot \left(\frac{2 \ln(x) + 2}{x} \right)$$

$$f'(x) = 0 \text{ geeft } e^{\ln^2(x) + 2 \ln(x) - 3} \cdot \left(\frac{2 \ln(x) + 2}{x} \right) = 0$$

$$\frac{2 \ln(x) + 2}{x} = 0$$

$$2 \ln(x) + 2 = 0$$

$$\ln(x) = -1$$

$$x = e^{-1}$$

$$\text{min. is } f(e^{-1}) = e^{(-1)^2 + 2 \cdot (-1) - 3} = e^{-4} = \frac{1}{e^4}, \text{ dus } B_f = \left[\frac{1}{e^4}, \rightarrow \right).$$

86 a $f(x) = x^x = (e^{\ln(x)})^x = e^{x \ln(x)}$ geeft $f'(x) = e^{x \ln(x)} \cdot \left(1 \cdot \ln(x) + x \cdot \frac{1}{x} \right) = (\ln(x) + 1) \cdot x^x$

b $g_a(x) = x^{\frac{a}{x}} = (e^{\ln(x)})^{\frac{a}{x}} = e^{\frac{a}{x} \ln(x)} = e^{ax^{-1} \cdot \ln(x)}$ geeft

$$g'_a(x) = e^{ax^{-1} \cdot \ln(x)} \cdot \left(-ax^{-2} \cdot \ln(x) + ax^{-1} \cdot \frac{1}{x} \right) = x^{\frac{a}{x}} \cdot \left(\frac{-a \ln(x)}{x^2} + \frac{a}{x^2} \right) = \left(\frac{a - a \ln(x)}{x^2} \right) \cdot x^{\frac{a}{x}}$$

Voor raken geldt $f(x) = g_a(x) \wedge f'(x) = g'_a(x)$.

$$f(x) = g_a(x) \text{ geeft } x^x = x^{\frac{a}{x}}$$

$$x = \frac{a}{x}$$

$$a = x^2$$

$$f'(x) = g'_a(x) \text{ met } a = x^2 \text{ geeft } (\ln(x) + 1) \cdot x^x = \left(\frac{x^2 - x^2 \ln(x)}{x^2} \right) \cdot x^{\frac{x^2}{x}}$$

$$(\ln(x) + 1) \cdot x^x = \left(\frac{x^2(1 - \ln(x))}{x^2} \right) \cdot x^x$$

$$\ln(x) + 1 = 1 - \ln(x)$$

$$2 \ln(x) = 0$$

$$x = 1$$

Dus $a = 1^2 = 1$.

Stel $k: y = mx + n$ met $m = f'(1) = (\ln(1) + 1) \cdot 1^1 = 1$.

$$\left. \begin{array}{l} y = x + n \\ f(1) = 1, \text{ dus } A(1, 1) \end{array} \right\} \begin{array}{l} 1 + n = 1 \\ n = 0 \end{array}$$

Dus $k: y = x$.

Diagnostische toets

Bladzijde 46

1 a ${}^2\log(18) - {}^2\log(36) + 3 = {}^2\log\left(\frac{18}{36}\right) + 3 = {}^2\log\left(\frac{1}{2}\right) + 3 = -1 + 3 = 2$

b $2 \cdot {}^3\log(5) - {}^3\log(75) + {}^2\log(8\sqrt{2}) = {}^3\log(5^2) - {}^3\log(75) + {}^2\log(2^3 \cdot 2^{\frac{1}{2}}) = {}^3\log(25) - {}^3\log(75) + {}^2\log(2^{3\frac{1}{2}}) = {}^3\log\left(\frac{25}{75}\right) + 3\frac{1}{2} = {}^3\log\left(\frac{1}{3}\right) + 3\frac{1}{2} = -1 + 3\frac{1}{2} = 2\frac{1}{2}$

2 a $4 + {}^2\log(a) + \log(100) = 4 + {}^2\log(a) + 2 = 6 + {}^2\log(a) = {}^2\log(2^6) + {}^2\log(a) = {}^2\log(64) + {}^2\log(a) = {}^2\log(64a)$

b $2 - {}^3\log(6a) = {}^3\log(3^2) - {}^3\log(6a) = {}^3\log(9) - {}^3\log(6a) = {}^3\log\left(\frac{9}{6a}\right) = {}^3\log\left(\frac{3}{2a}\right)$

3 a $3 \cdot {}^3\log(36) = 3 \cdot {}^3\log(9 \cdot 4) = 3 \cdot ({}^3\log(9) + {}^3\log(4)) = 3 \cdot (2 + {}^3\log(4)) = 6 + 3 \cdot {}^3\log(4) = 6 + {}^3\log(4^3) = 6 + {}^3\log(64)$

b $3 + \log(800) = 3 + \log(100 \cdot 8) = 3 + \log(100) + \log(8) = 3 + 2 + \log(8) = 5 + \log(8)$

- 4 a** $y = {}^5\log(x) \xrightarrow{\text{translatie } (0, 2)} y = {}^5\log(x) + 2$
 Er geldt ${}^5\log(x) + 2 = {}^5\log(x) + {}^5\log(5^2) = {}^5\log(x) + {}^5\log(25) = {}^5\log(25x)$.
 Dus de vermenigvuldiging ten opzichte van de y -as met $\frac{1}{25}$ levert dezelfde beeldgrafiek op.
- b** $y = {}^3\log(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = {}^3\log(3x)$
 Er geldt ${}^3\log(3x) = {}^3\log(3) + {}^3\log(x) = 1 + {}^3\log(x)$.
 Dus de translatie $(0, 1)$ levert dezelfde beeldgrafiek op.

- 5 a** $2 \cdot {}^2\log(x-1) = 1 + {}^2\log(18)$
 ${}^2\log((x-1)^2) = {}^2\log(2) + {}^2\log(18)$
 ${}^2\log((x-1)^2) = {}^2\log(2 \cdot 18)$
 ${}^2\log((x-1)^2) = {}^2\log(36)$
 $(x-1)^2 = 36$
 $x-1 = 6 \vee x-1 = -6$
 $x = 7 \vee x = -5$
 vold. niet
- b** ${}^3\log(2x-1) + \frac{1}{3}\log(x+2) = 0$
 ${}^3\log(2x-1) - {}^3\log(x+2) = 0$
 ${}^3\log(2x-1) = {}^3\log(x+2)$
 $2x-1 = x+2$
 $x = 3$
- c** ${}^2\log(2x-1) = {}^4\log(x)$
 ${}^2\log(2x-1) = \frac{{}^2\log(x)}{{}^2\log(4)}$
 ${}^2\log(2x-1) = \frac{{}^2\log(x)}{2}$
 $2 \cdot {}^2\log(2x-1) = {}^2\log(x)$
 ${}^2\log((2x-1)^2) = {}^2\log(x)$
 $(2x-1)^2 = x$
 $4x^2 - 4x + 1 = x$
 $4x^2 - 5x + 1 = 0$
 $D = (-5)^2 - 4 \cdot 4 \cdot 1 = 9$
 $x = \frac{5 \pm 3}{8} = 1 \vee x = \frac{5-3}{8} = \frac{1}{4}$
 vold. niet
- d** $\log^2(x) = \log(x) + 2$
 Stel $\log(x) = u$.
 $u^2 = u + 2$
 $u^2 - u - 2 = 0$
 $(u+1)(u-2) = 0$
 $u = -1 \vee u = 2$
 $\log(x) = -1 \vee \log(x) = 2$
 $x = \frac{1}{10} \vee x = 100$
- e** ${}^2\log(x) = 3 - {}^2\log(x+2)$
 ${}^2\log(x) = {}^2\log(8) - {}^2\log(x+2)$
 ${}^2\log(x) + {}^2\log(x+2) = {}^2\log(8)$
 ${}^2\log(x(x+2)) = {}^2\log(8)$
 $x(x+2) = 8$
 $x^2 + 2x - 8 = 0$
 $(x-2)(x+4) = 0$
 $x = 2 \vee x = -4$
 vold. niet
- f** ${}^2\log^2(x) + 12 = 7 \cdot {}^2\log(x)$
 Stel ${}^2\log(x) = u$.
 $u^2 + 12 = 7u$
 $u^2 - 7u + 12 = 0$
 $(u-3)(u-4) = 0$
 $u = 3 \vee u = 4$
 ${}^2\log(x) = 3 \vee {}^2\log(x) = 4$
 $x = 8 \vee x = 16$

- 6 a** $g_{\text{dag}} = 1,1$
 $g_{\text{week}} = 1,1^7 \approx 1,949$
 Het groeipercentage per week is 94,9%.
- b** $g_{\text{dag}} = 1,1$
 $g_{8 \text{ uur}} = 1,1^{\frac{1}{3}} \approx 1,032$
 Het groeipercentage per 8 uur is 3,2%.

- 7 a** $g_{\text{jaar}} = 0,64$
 $g_{\text{maand}} = 0,64^{\frac{1}{12}} \approx 0,963$
 De afname per maand is 3,7%.
- b** $g_{\text{jaar}} = 0,64$
 $g_{5 \text{ jaar}} = 0,64^5 \approx 0,107$
 De afname per 5 jaar is 89,3%.

- 8 a** $g_{\text{maand}} = 1,002$
 $1,002^T = 2$
 $T = {}^{1,002}\log(2) = 346,920\dots$
 De verdubbelingstijd is $346,920\dots : 12 \approx 29$ jaar.
- b** $g_{\text{week}} = 0,8$
 $0,8^T = \frac{1}{2}$
 $T = {}^{0,8}\log(\frac{1}{2}) = 3,106\dots$
 De halveringstijd is $3,106\dots \cdot 7 \approx 22$ dagen.

- 9 a** $y = 4 \cdot \log(3x + 12) - 1$
 $4 \cdot \log(3x + 12) = y + 1$
 $\log(3x + 12) = 0,25y + 0,25$
 $3x + 12 = 10^{0,25y + 0,25}$
 $3x + 12 = 10^{0,25y} \cdot 10^{0,25}$
 $3x = -12 + (10^{0,25})^y \cdot 10^{0,25}$
 $x \approx -4 + 0,59 \cdot 1,78^y$
 Dus $x = -4 + 0,59 \cdot 1,78^y$.
- b** $p = 34 \cdot 2,7^{5q+2}$
 $34 \cdot 2,7^{5q+2} = p$
 $2,7^{5q+2} = \frac{1}{34}p$
 $5q + 2 = {}^{2,7}\log(\frac{1}{34}p)$
 $5q + 2 = {}^{2,7}\log(\frac{1}{34}) + {}^{2,7}\log(p)$
 $5q = -2 + \frac{\log(\frac{1}{34})}{\log(2,7)} + \frac{\log(p)}{\log(2,7)}$
 $q = -\frac{2}{5} + \frac{\log(\frac{1}{34})}{5\log(2,7)} + \frac{\log(p)}{5\log(2,7)}$
 $q \approx -1,11 + 0,46 \cdot \log(p)$
 Dus $q = -1,11 + 0,46 \cdot \log(p)$.

Bladzijde 47

- 10 a** $\frac{3e^3 - e^3}{e^2} = \frac{2e^3}{e^2} = 2e$
- b** $\frac{e^{3x} - e^x}{e^x} = e^{2x} - 1$
- c** $(e^{3x} - 5)^2 = (e^{3x})^2 - 2 \cdot e^{3x} \cdot 5 + 25 = e^{6x} - 10e^{3x} + 25$
- 11 a** $3xe^x - e^x = 0$
 $e^x(3x - 1) = 0$
 $3x - 1 = 0$
 $3x = 1$
 $x = \frac{1}{3}$
- b** $e^{2x-1} - \sqrt[3]{e^2} = 0$
 $e^{2x-1} = \sqrt[3]{e^2}$
 $e^{2x-1} = e^{\frac{2}{3}}$
 $2x - 1 = \frac{2}{3}$
 $2x = 1\frac{2}{3}$
 $x = \frac{5}{6}$
- c** $e^{2x} + 2e^x = 3$
 $(e^x)^2 + 2e^x - 3 = 0$
 Stel $e^x = u$.
 $u^2 + 2u - 3 = 0$
 $(u - 1)(u + 3) = 0$
 $u = 1 \vee u = -3$
 $e^x = 1 \vee e^x = -3$
 $x = 0$

- 12 a** $f(x) = 2e^x - 3x^2$ geeft $f'(x) = 2e^x - 6x$
- b** $f(x) = \frac{x^2 + 1}{e^x}$ geeft $f'(x) = \frac{e^x \cdot 2x - (x^2 + 1) \cdot e^x}{(e^x)^2} = \frac{-x^2 + 2x - 1}{e^x}$
- c** $f(x) = (x^2 + 1)e^x$ geeft $f'(x) = 2x \cdot e^x + (x^2 + 1) \cdot e^x = (x^2 + 2x + 1)e^x$
- d** $f(x) = \frac{e^x}{x^2 + 1}$ geeft $f'(x) = \frac{(x^2 + 1) \cdot e^x - e^x \cdot 2x}{(x^2 + 1)^2} = \frac{(x^2 - 2x + 1)e^x}{(x^2 + 1)^2}$
- e** $f(x) = x^2 \cdot e^{2x-1}$ geeft $f'(x) = 2x \cdot e^{2x-1} + x^2 \cdot e^{2x-1} \cdot 2 = (2x^2 + 2x)e^{2x-1}$
- f** $f(x) = e^{x^2+9}$ geeft $f'(x) = e^{x^2+9} \cdot 2x = 2xe^{x^2+9}$

- 13 a** $f(x) = (x^2 - x + 1)e^x$ geeft $f'(x) = (2x - 1) \cdot e^x + (x^2 - x + 1) \cdot e^x = (x^2 + x)e^x$
 $f'(x) = 0$ geeft $(x^2 + x)e^x = 0$
 $x^2 + x = 0$
 $x(x + 1) = 0$
 $x = 0 \vee x = -1$

max. is $f(-1) = \frac{3}{e}$ en min. is $f(0) = 1$.

- b** $f'(x) = (x^2 + x)e^x$ geeft $f''(x) = (2x + 1) \cdot e^x + (x^2 + x) \cdot e^x = (x^2 + 3x + 1)e^x$
 $f''(x) = 0$ geeft $(x^2 + 3x + 1)e^x = 0$
 $x^2 + 3x + 1 = 0$
 $D = 3^2 - 4 \cdot 1 \cdot 1 = 5$
 $x = \frac{-3 \pm \sqrt{5}}{2} \vee x = \frac{-3 - \sqrt{5}}{2}$
 $x = -1\frac{1}{2} + \frac{1}{2}\sqrt{5} \vee x = -1\frac{1}{2} - \frac{1}{2}\sqrt{5}$

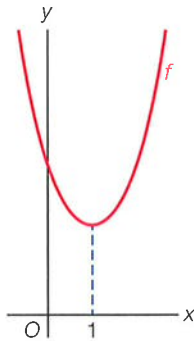
- c** Stel $k: y = ax + b$ met $a = f'(1) = (1 + 1)e = 2e$.
 $\left. \begin{array}{l} k: y = 2ex + b \\ f(1) = e, \text{ dus } A(1, e) \end{array} \right\} \begin{array}{l} 2e + b = e \\ b = -e \end{array}$
Dus $k: y = 2ex - e$.

- 14 a** $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$
- b** $\ln(10) - 4\ln(2) = \ln(10) - \ln(2^4) = \ln(10) - \ln(16) = \ln(\frac{10}{16}) = \ln(\frac{5}{8})$

- 15 a** $2e^{5x} = 16$
 $e^{5x} = 8$
 $5x = \ln(8)$
 $x = \frac{1}{5}\ln(8)$
- b** $\ln^2(5x) = 16$
 $\ln(5x) = 4 \vee \ln(5x) = -4$
 $5x = e^4 \vee 5x = e^{-4}$
 $x = \frac{1}{5}e^4 \vee x = \frac{1}{5e^4}$
- c** $2\ln^2(x) - \ln(x) = 0$
 $\ln(x)(2\ln(x) - 1) = 0$
 $\ln(x) = 0 \vee 2\ln(x) - 1 = 0$
 $x = 1 \vee 2\ln(x) = 1$
 $x = 1 \vee \ln(x) = \frac{1}{2}$
 $x = 1 \vee x = e^{\frac{1}{2}} = \sqrt{e}$
- d** $\ln(9x + 1) - \ln(x + 2) = \ln(4)$
 $\ln(9x + 1) = \ln(x + 2) + \ln(4)$
 $\ln(9x + 1) = \ln(4(x + 2))$
 $9x + 1 = 4(x + 2)$
 $9x + 1 = 4x + 8$
 $5x = 7$
 $x = 1\frac{2}{5}$

- 16 a** $f(x) = 2^{3x-4}$ geeft $f'(x) = 2^{3x-4} \cdot \ln(2) \cdot 3 = 3 \cdot 2^{3x-4} \cdot \ln(2)$
- b** $f(x) = x \cdot 3^x$ geeft $f'(x) = 1 \cdot 3^x + x \cdot 3^x \cdot \ln(3) = (1 + x\ln(3)) \cdot 3^x$
- c** $f(x) = \ln(x \cdot \sqrt[3]{x}) = \ln(x^{1\frac{1}{3}}) = \frac{1}{3}\ln(x)$ geeft $f'(x) = \frac{1}{3} \cdot \frac{1}{x} = \frac{1}{3x}$
- d** $f(x) = {}^2\log(4x) = {}^2\log(4) + {}^2\log(x)$ geeft $f'(x) = \frac{1}{x\ln(2)}$
- e** $f(x) = {}^3\log(5x - 6)$ geeft $f'(x) = \frac{1}{(5x - 6)\ln(3)} \cdot 5 = \frac{5}{(5x - 6)\ln(3)}$
- f** $f(x) = \ln(3x^2 + 3)$ geeft $f'(x) = \frac{1}{3x^2 + 3} \cdot 6x = \frac{2x}{x^2 + 1}$

17 $f(x) = 3^{x-1} + 3^{-x+1}$ geeft $f'(x) = 3^{x-1} \cdot \ln(3) + 3^{-x+1} \cdot \ln(3) \cdot -1 = (3^{x-1} - 3^{-x+1}) \ln(3)$
 $f'(x) = 0$ geeft $(3^{x-1} - 3^{-x+1}) \ln(3) = 0$
 $3^{x-1} - 3^{-x+1} = 0$
 $3^{x-1} = 3^{-x+1}$
 $x - 1 = -x + 1$
 $2x = 2$
 $x = 1$



min. is $f(1) = 3^0 + 3^0 = 1 + 1 = 2$, dus $B_f = [2, \rightarrow)$.

18 a $f(x) = x^2 \ln(x) - \frac{1}{2}x^2$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x} - x = 2x \ln(x)$
 $f'(x) = 0$ geeft $2x \ln(x) = 0$
 $x = 0 \vee \ln(x) = 0$
 $x = 0 \vee x = 1$
vold. niet
 $f(1) = 1 \cdot 0 - \frac{1}{2} \cdot 1 = -\frac{1}{2}$, dus $A(1, -\frac{1}{2})$.

b $f'(x) = 2x \ln(x)$ geeft $f''(x) = 2 \cdot \ln(x) + 2x \cdot \frac{1}{x} = 2 \ln(x) + 2$
 $f''(x) = 0$ geeft $2 \ln(x) + 2 = 0$
 $2 \ln(x) = -2$
 $\ln(x) = -1$
 $x = e^{-1} = \frac{1}{e}$

Stel $k: y = ax + b$ met $a = f'\left(\frac{1}{e}\right) = 2 \cdot \frac{1}{e} \cdot -1 = -\frac{2}{e}$.

$$k: y = -\frac{2}{e}x + b \left\{ \begin{array}{l} -\frac{2}{e} \cdot \frac{1}{e} + b = -\frac{3}{2e^2} \\ -\frac{4}{2e^2} + b = -\frac{3}{2e^2} \end{array} \right.$$

$$b = \frac{1}{2e^2}$$

Dus $k: y = -\frac{2}{e}x + \frac{1}{2e^2}$.